EXPERTISE

POLITECNICO DI TORINO

Department of Mechanical and Aerospace Engineering

KICK-OFF TRAINING

TUM - Garching-bei-München

2018 Jan 8-12

Mistuning and damping devices

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Politecnico di Torino

Agenda

Introduction and mathematical models

- The effect of asymmetries in bladed disks
- Symptoms of mistuning and causes
- Lumped parameters model
- Numerical techniques to take into account mistuning: a literature overview
- Numerical techniques to take into account mistuning in the presence of friction contacts

The effect of asymmetries in bladed disks

Test rig design to study the dynamics of bladed disks

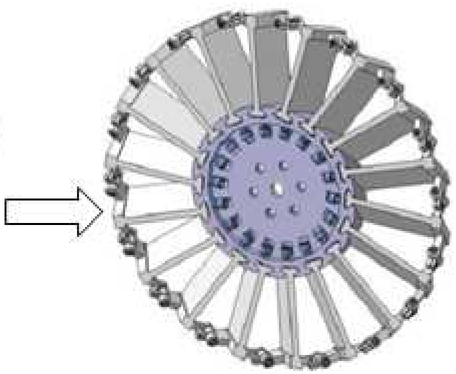
Requirement:

- Presence of joints: shroud and blade root joints
- Out-of-plane displacements
- LPT configuration & machining costs

Proposal:

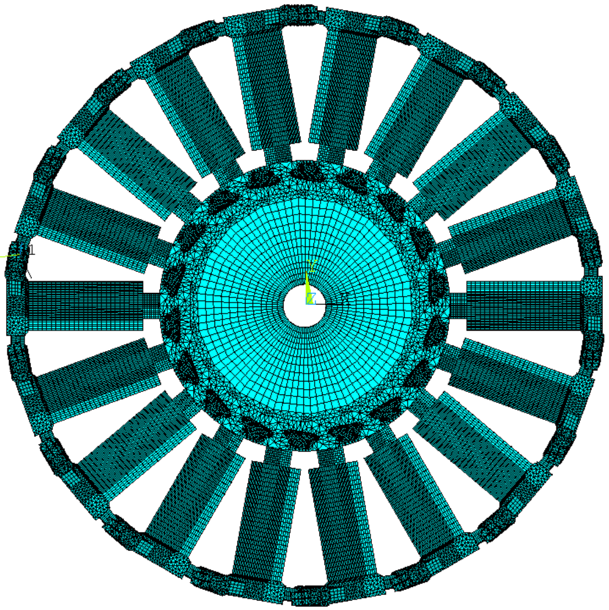
- Non-integral bladed disk
- 45° stagger angle
- Blade vibration at 1F resonances

real assembly



design of the full test rig

The final solution comes as an iterative process based on FEM

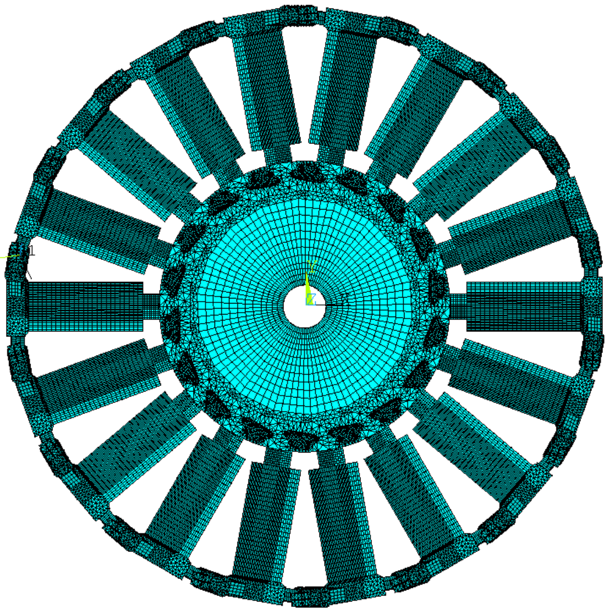


N=18 Blades

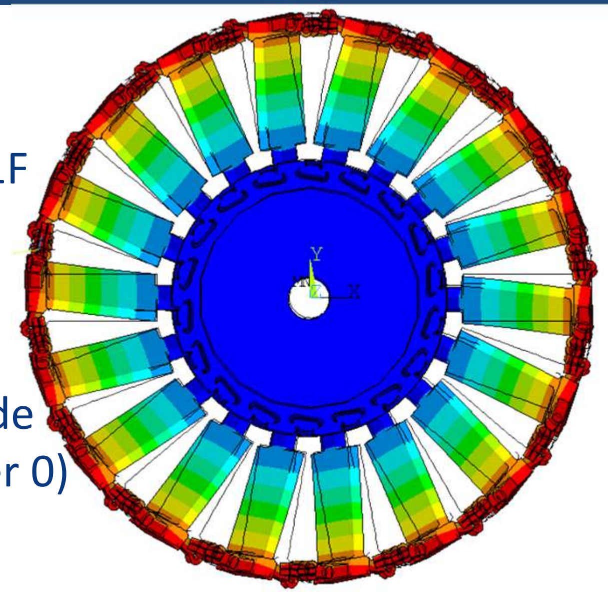
The effect of asymmetries in bladed disks

FEM modelling - Modal analysis

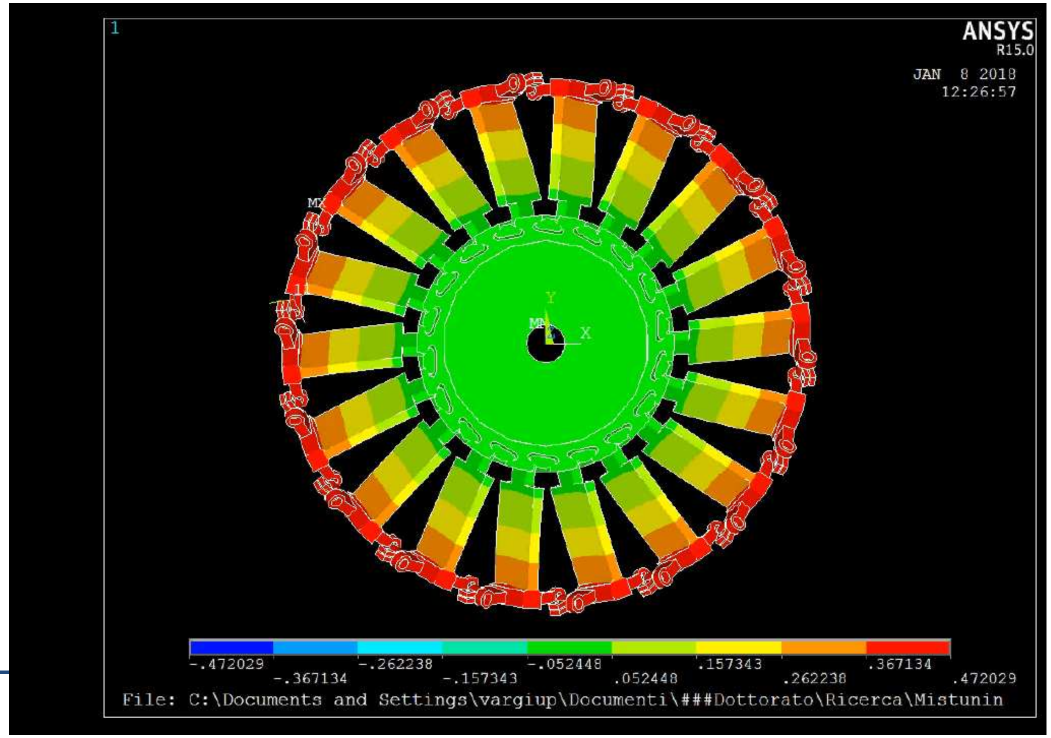
The final solution comes as an iterative process based on FEM



1F



Umbrella mode
(Nodal Diameter 0)
f_0=147 Hz



FEM modelling - Modal analysis

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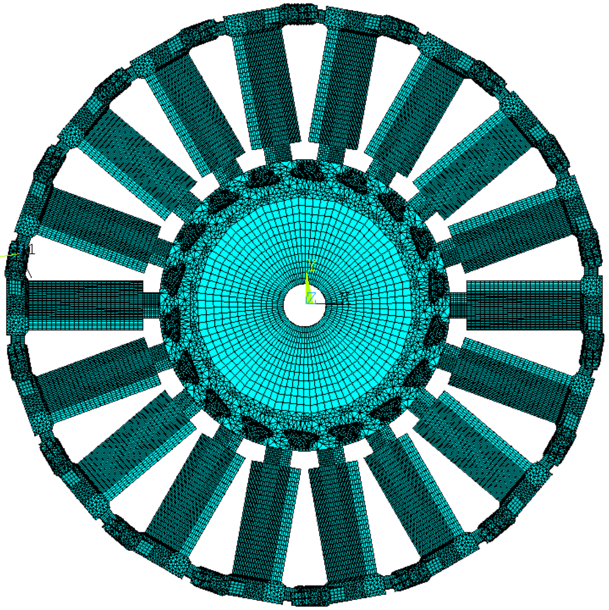

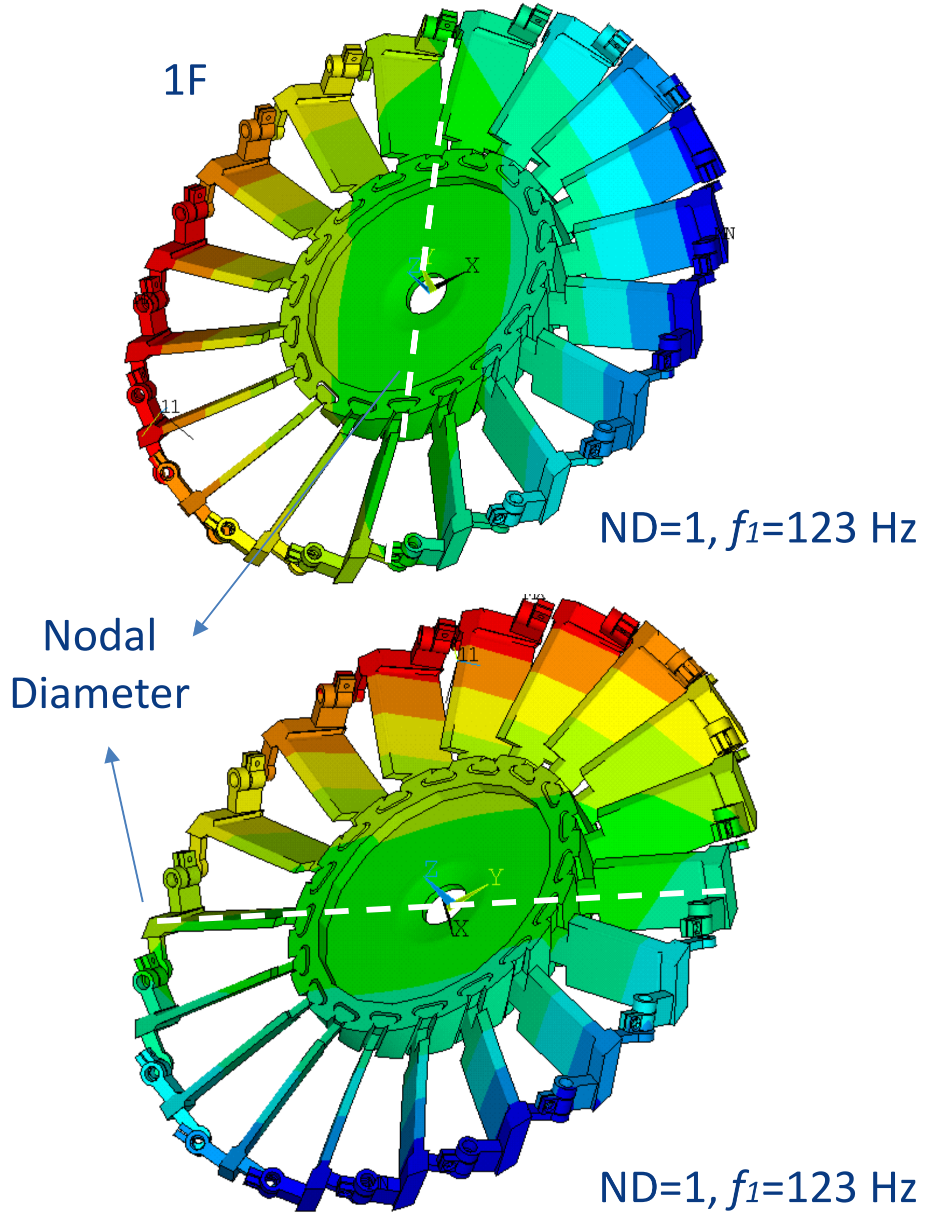

FEM modelling - Modal analysis

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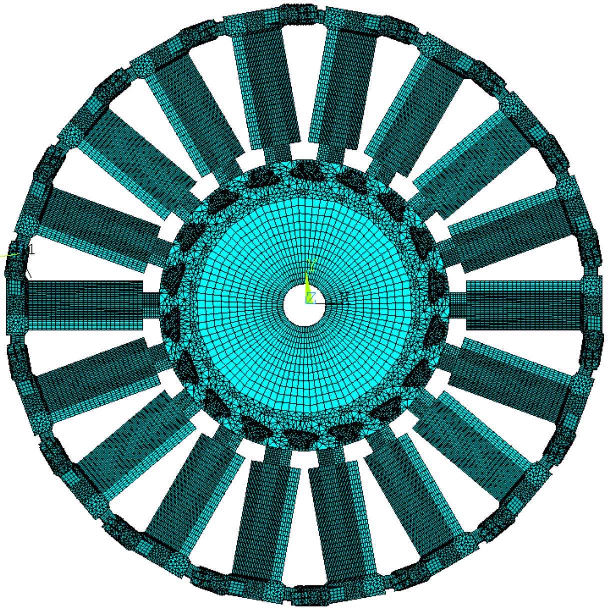
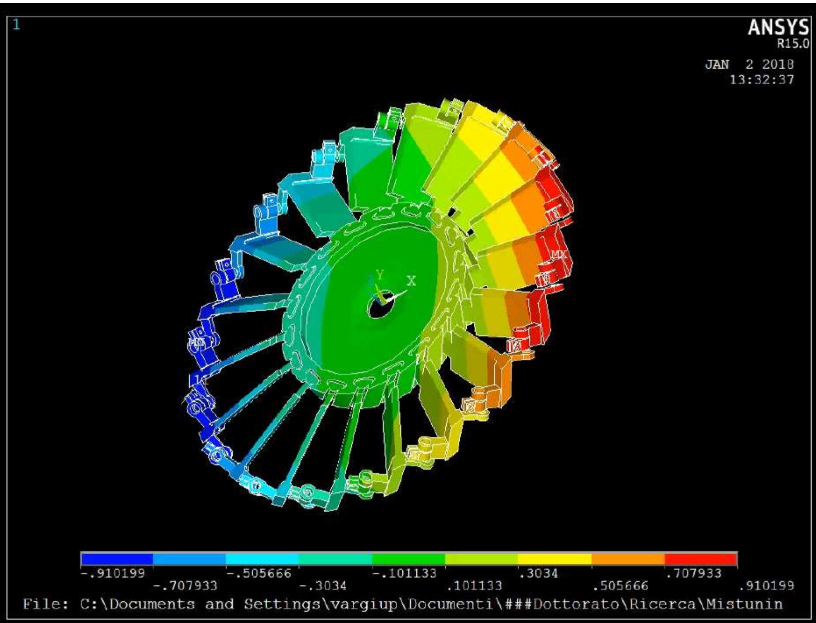
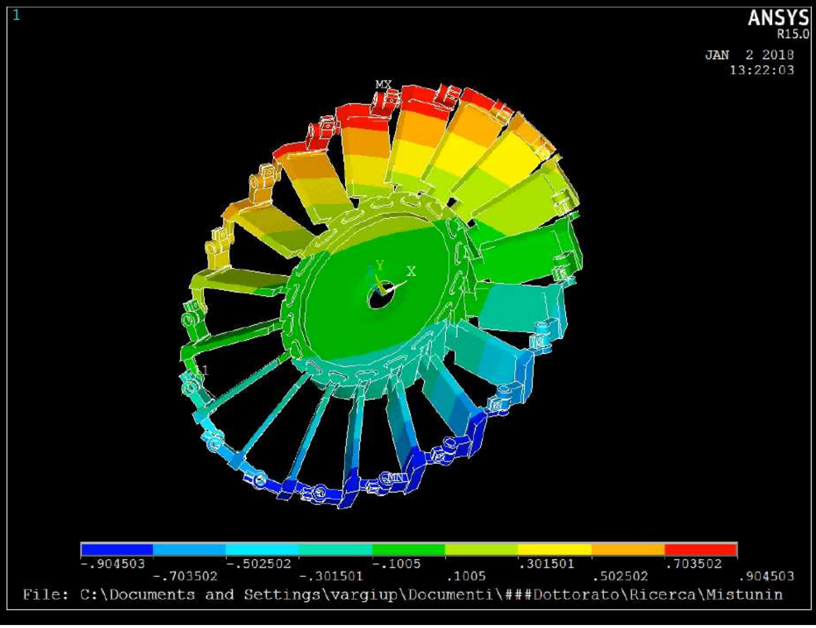


ND=1,
f_1=123 Hz

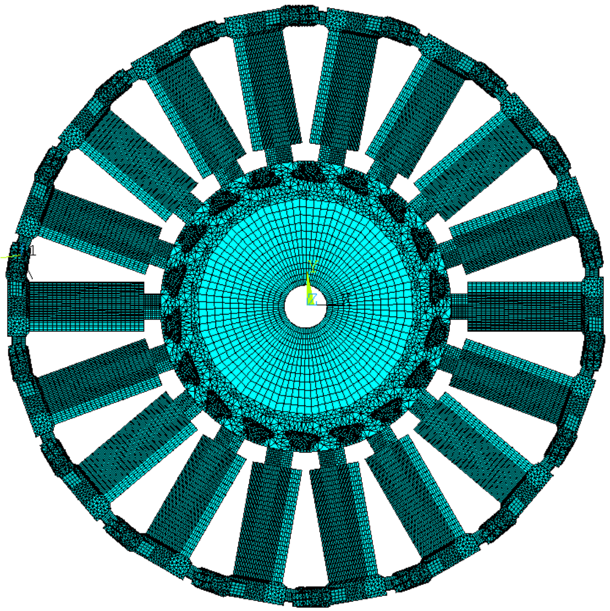


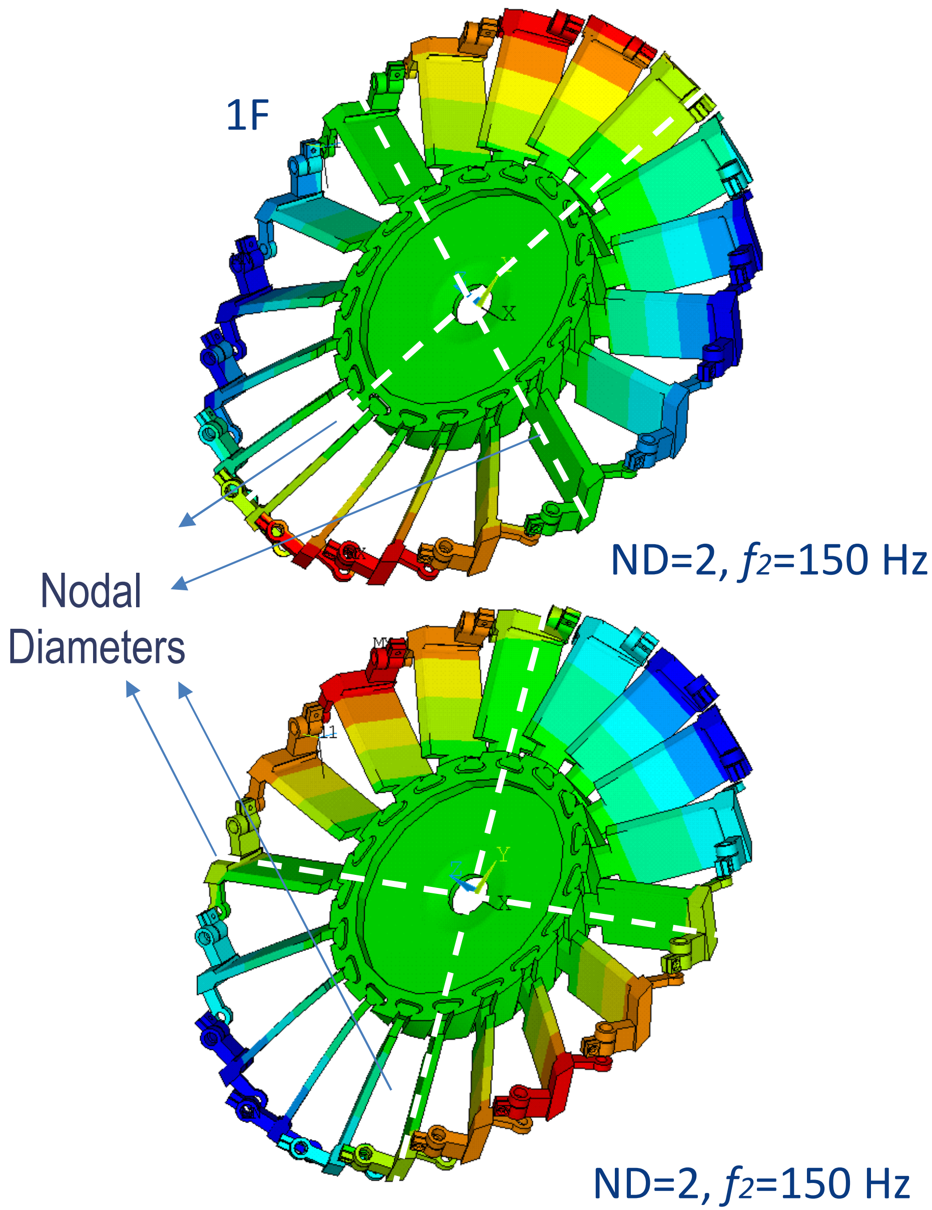
ND=1,
f_1=123 Hz



FEM modelling - Modal analysis

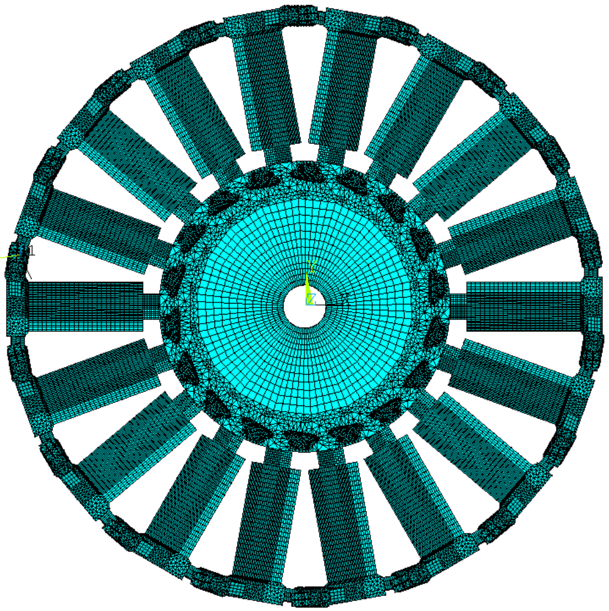
The final solution comes as an iterative process based on FEM



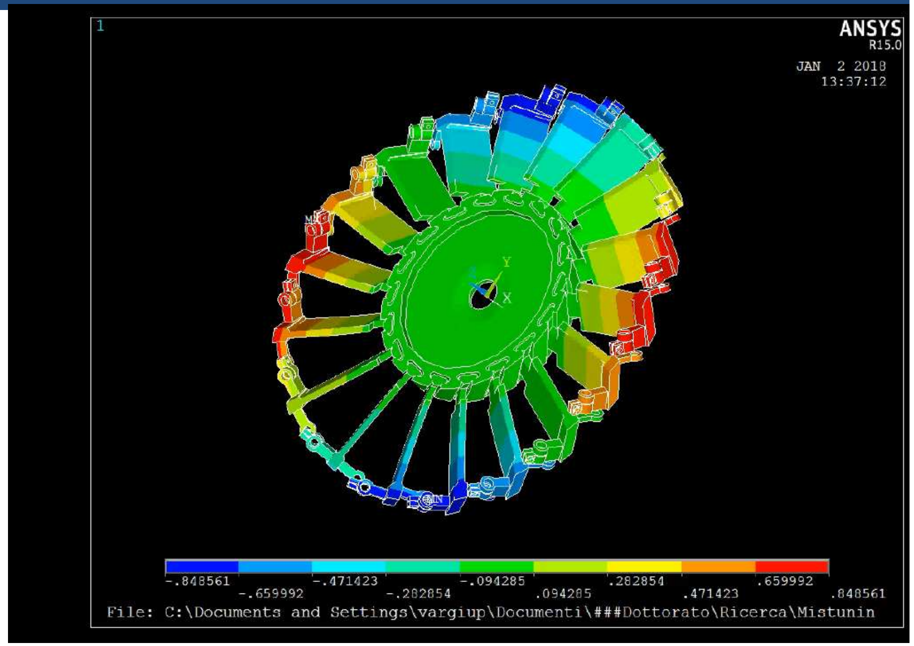


FEM modelling - Modal analysis

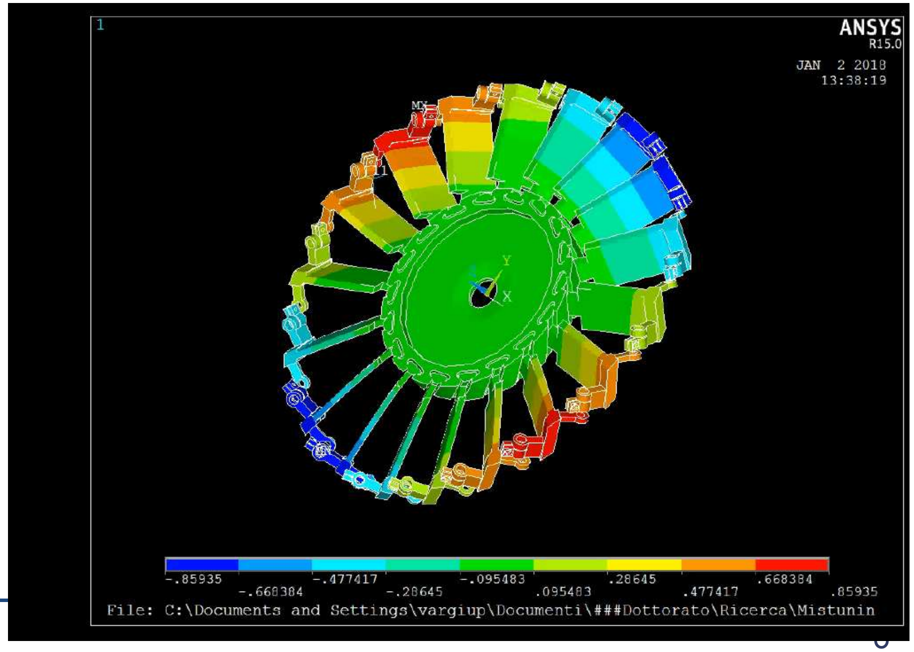
The final solution comes as an iterative process based on FEM



ND=2,
f_2=150 Hz



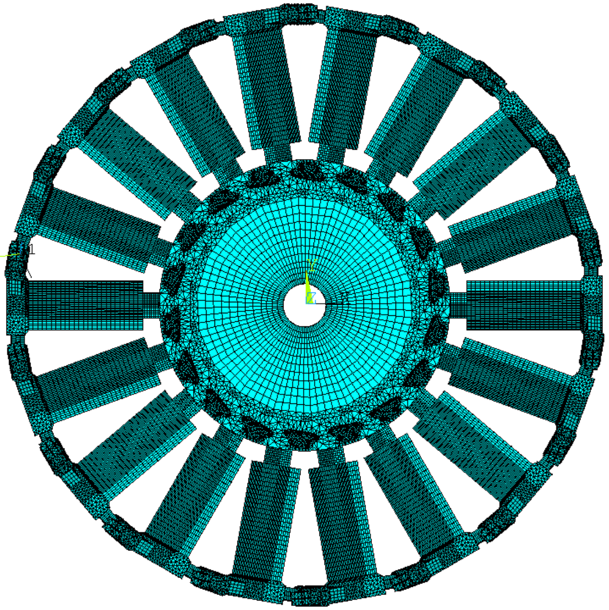
ND=2,
f_2=150 Hz



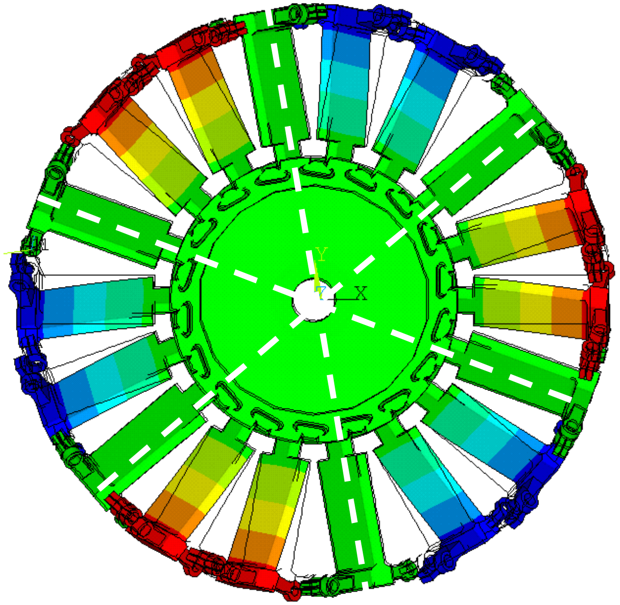
The effect of asymmetries in bladed disks

FEM modelling - Modal analysis

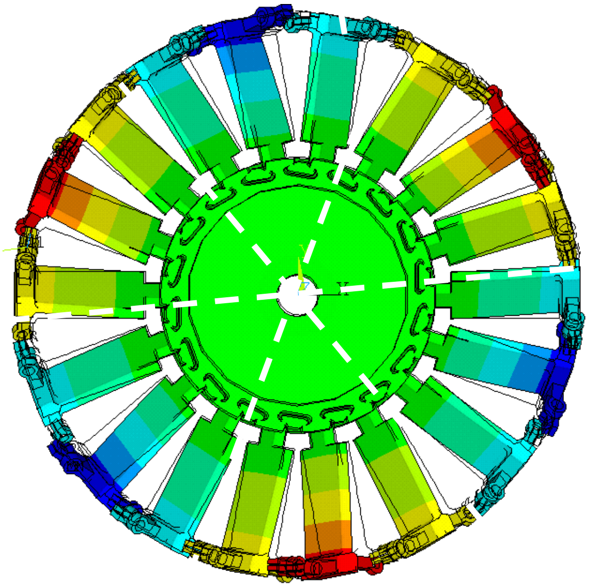
The final solution comes as an iterative process based on FEM



1F



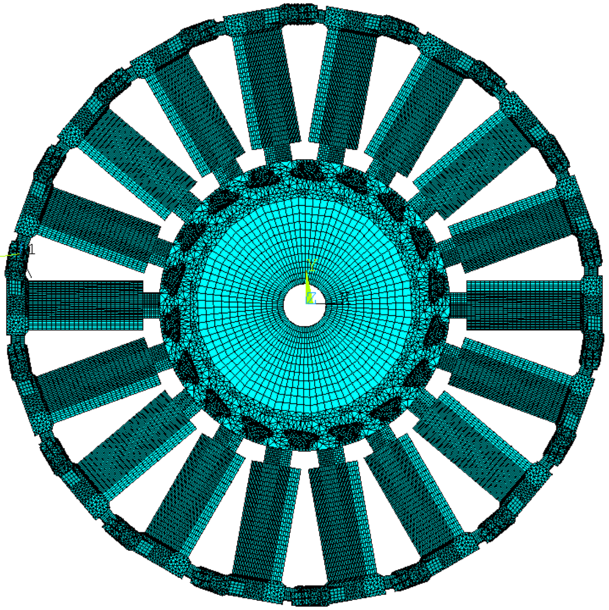
ND=3, f_3=155 Hz



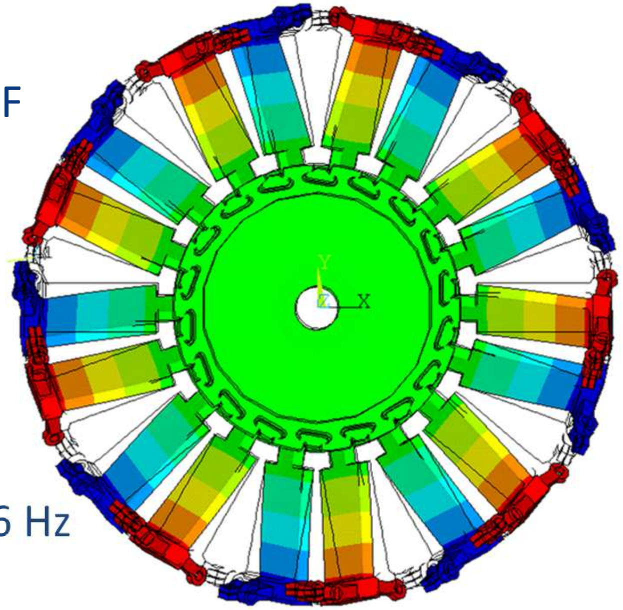
ND=3, f_3=155 Hz

FEM modelling - Modal analysis

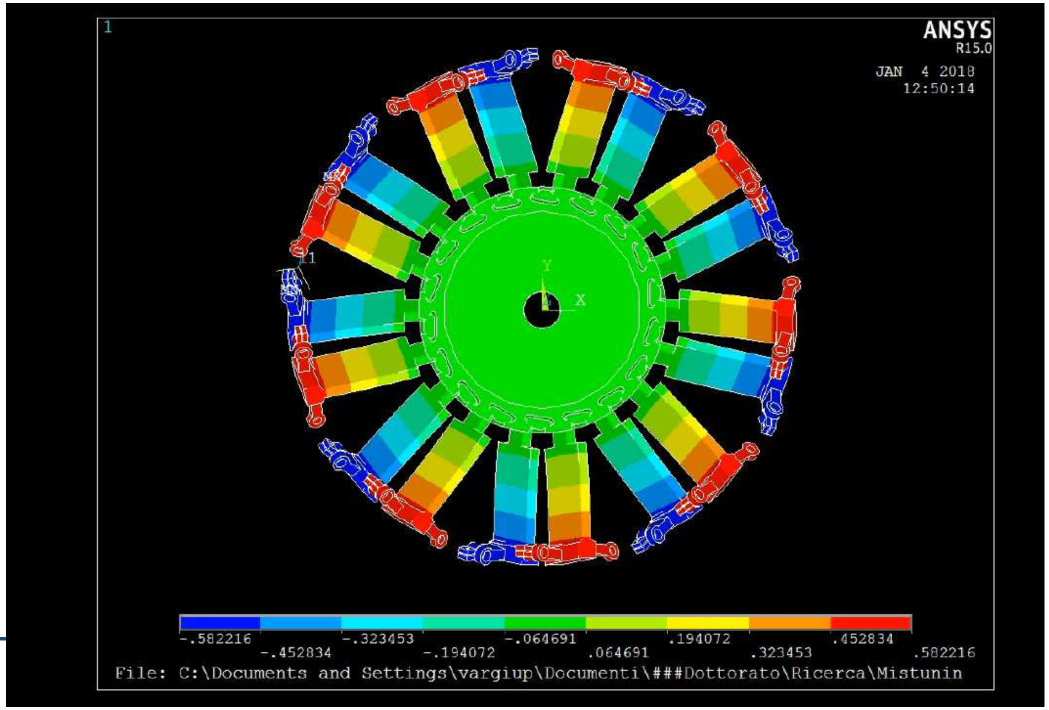
The final solution comes as an iterative process based on FEM



1F



…ND=9, f_9=156 Hz

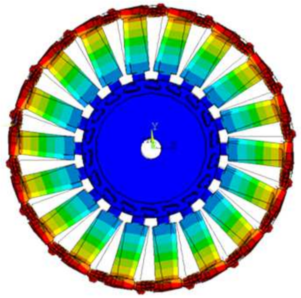


The effect of asymmetries in bladed disks

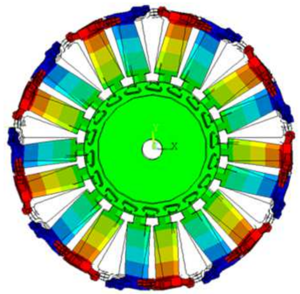
FEM modelling - Modal analysis

To resume: a modal family is obtained

- ND_{min}=0 and ND_{max}=int(N/2) are single modes and are stationary (real eigenvectors)

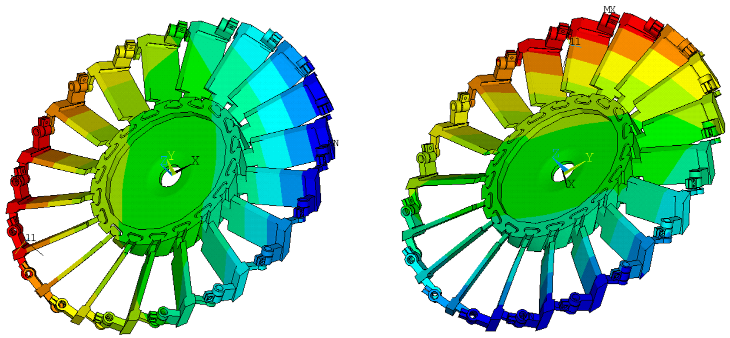


Umbrella mode
(ND=0)
f_0=147 Hz

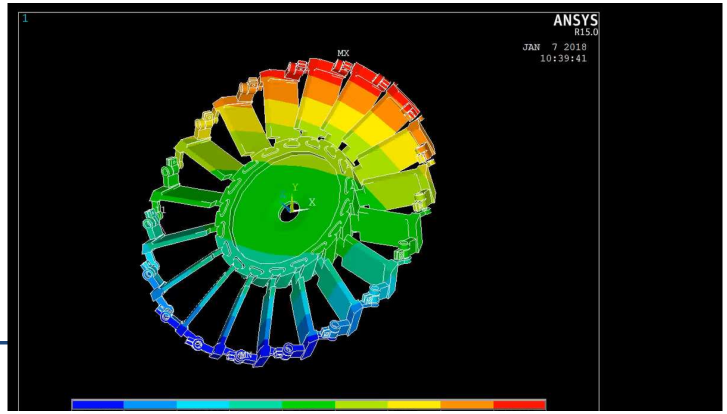


ND=9
f_9=156 Hz

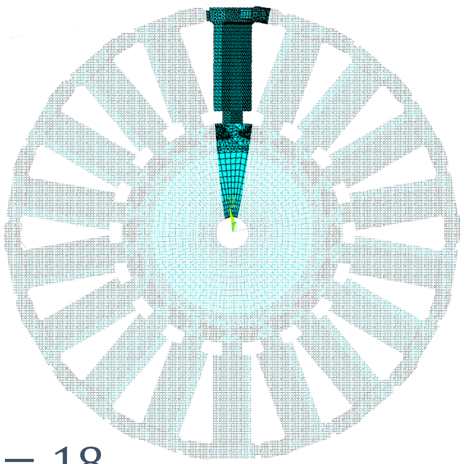
- The other ND mode shapes are repeated (stationary, real eigenvectors) and, if properly excited, give rotating mode shapes (complex eigenvectors)
  - Each sector behaves like the other with a given phase lag
  - I can use one basic sector to calculate the whole mode shape (*Cyclic symmetry constraint*)



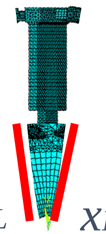
ND=1, f_1=123 Hz



FEM modelling - cyclic symmetry

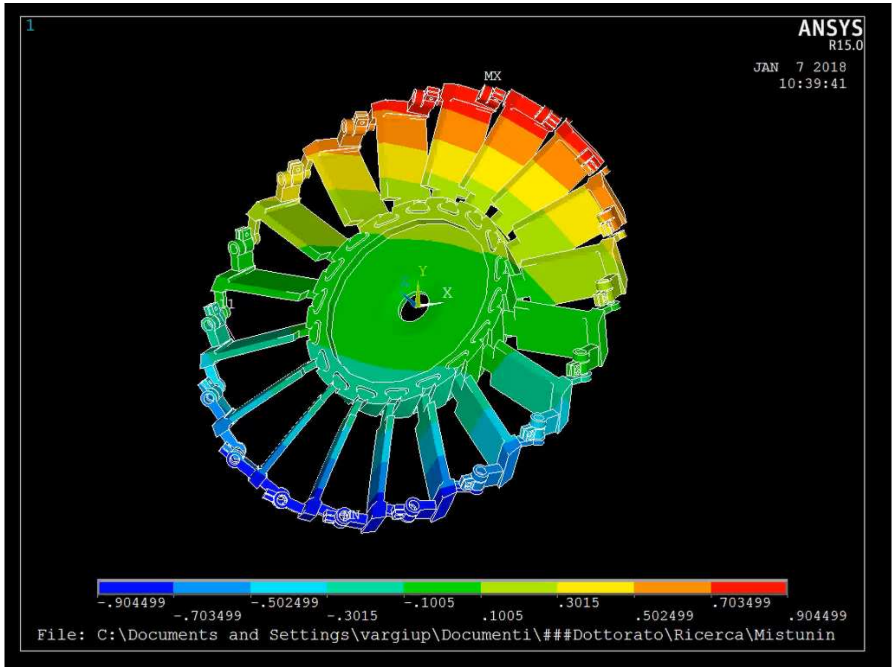


$N = 18$

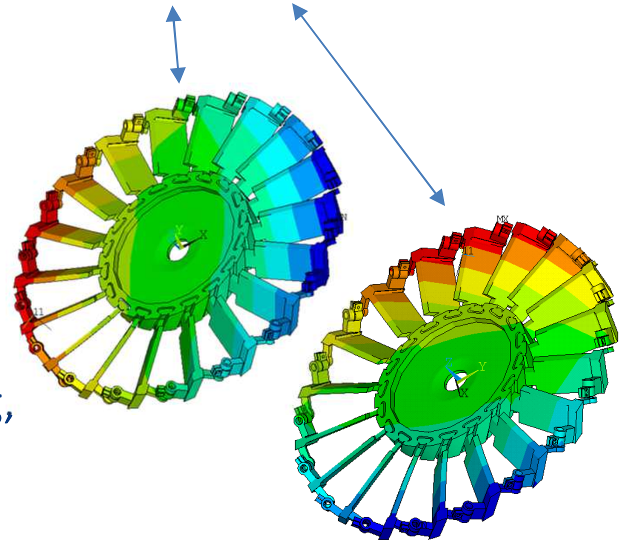


$$x_R = x_L e^{j\varphi}$$

IBPA: $\varphi = \frac{2\pi}{N} ND$



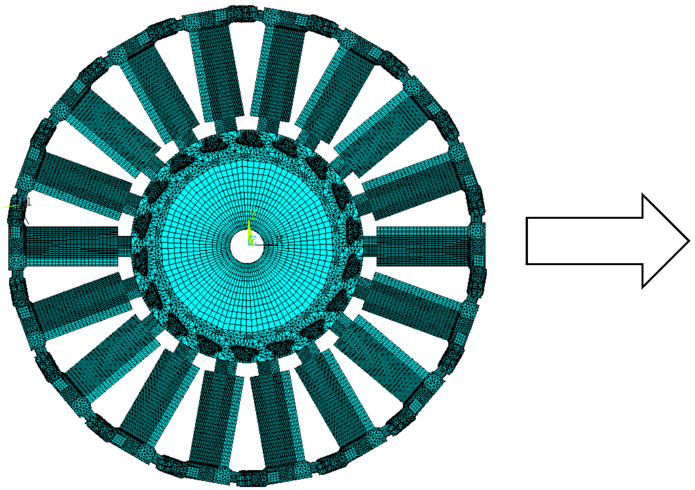
- If $\varphi = 0 \rightarrow x_R = x_L \rightarrow$ ND=0 stationary mode
- If $\varphi = \pi \rightarrow x_R = -x_L \rightarrow$ ND=9 stationary mode
- If $0 < \varphi < \pi \rightarrow$ 0 < ND < 9 rotating, complex modes
  <u>note</u>: the two stationary, repeated modes can be associated to the real and imaginary part of the rotating, complex mode



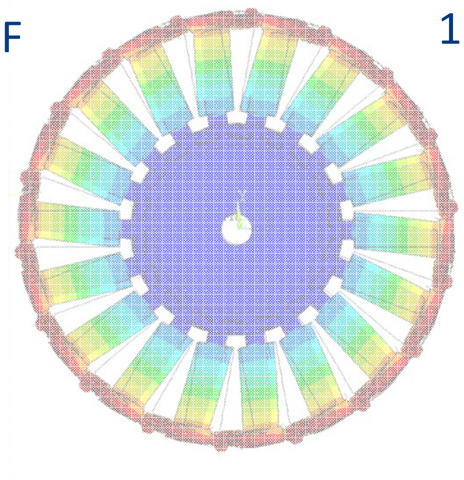
The effect of asymmetries in bladed disks

FEM modelling – modal families

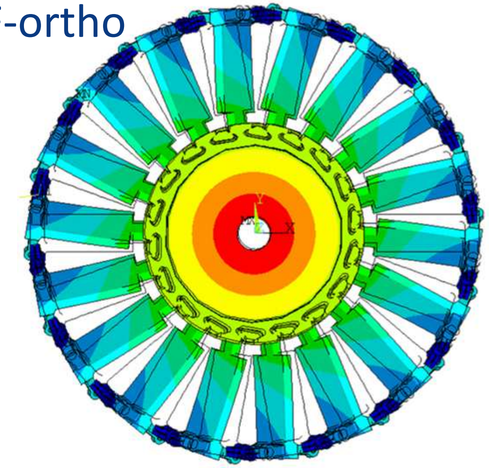
Different modal families exist, all of them characterized by the 0…$N/2$ ND sequence



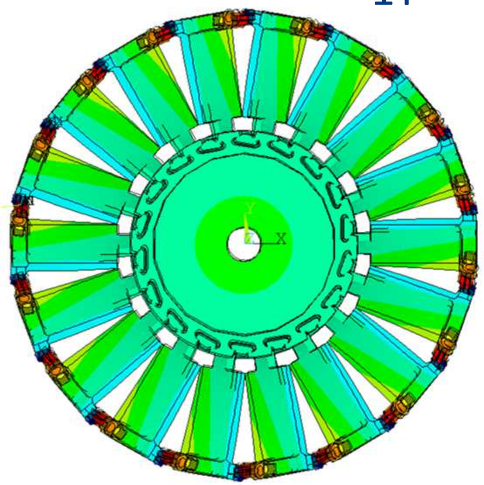
1F



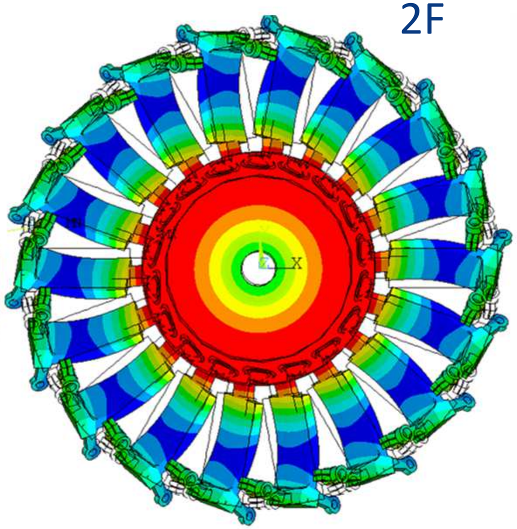
1F-ortho



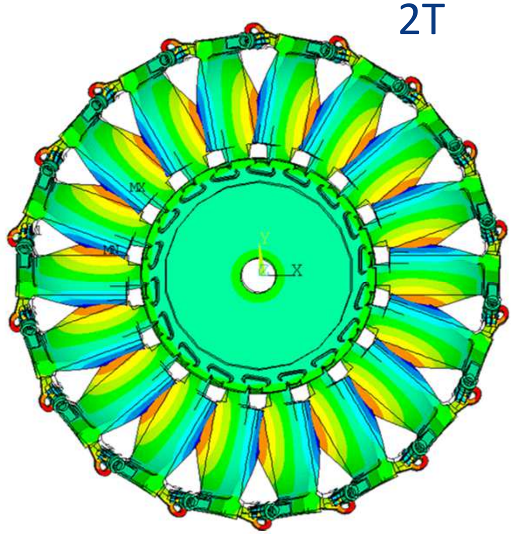
1T



2F

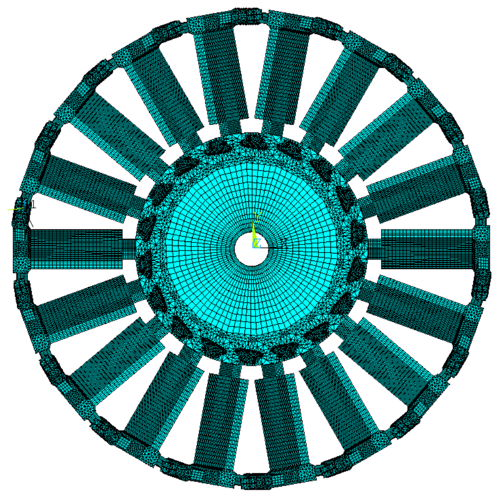


2T

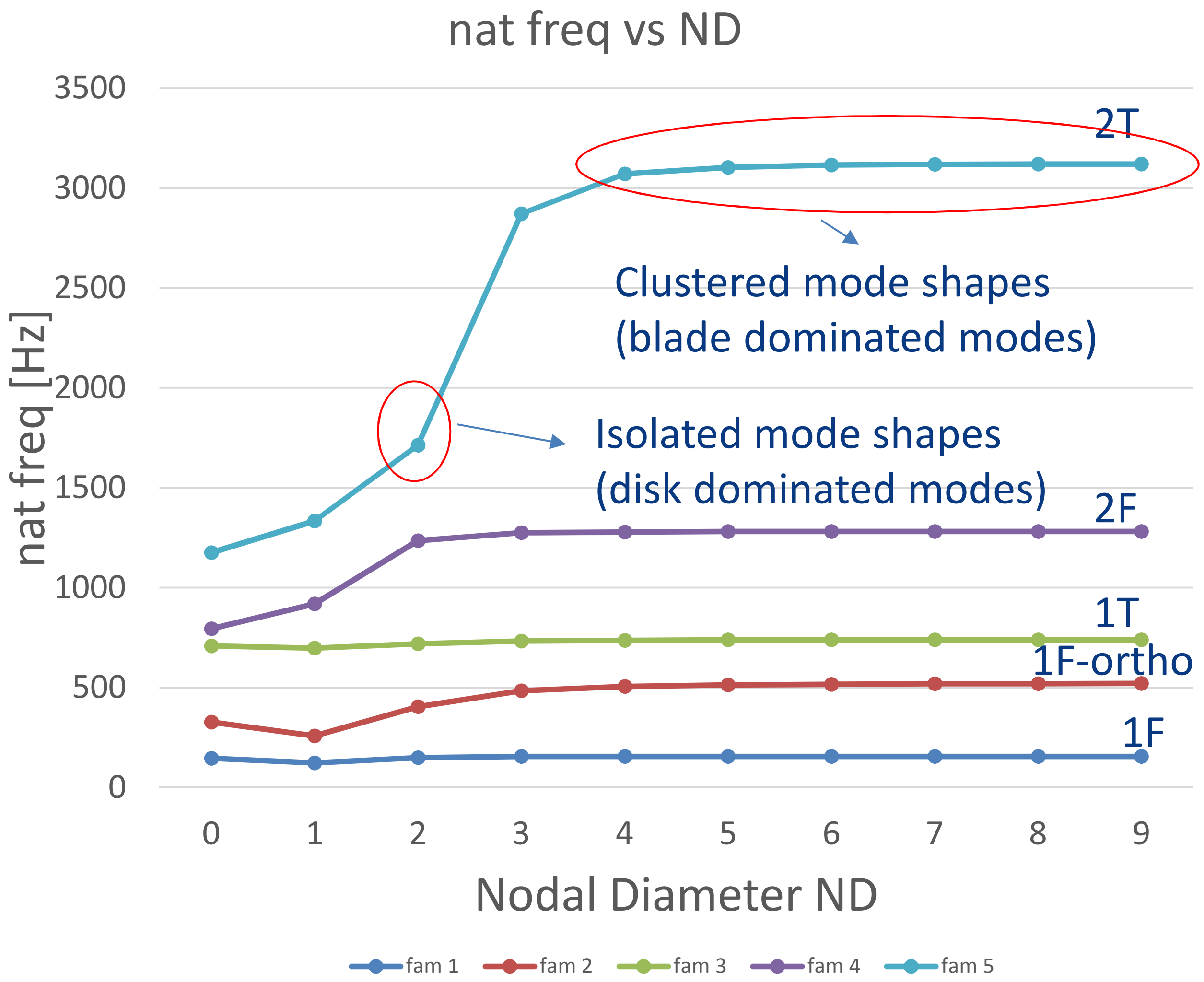


The effect of asymmetries in bladed disks

FEM modelling – modal families

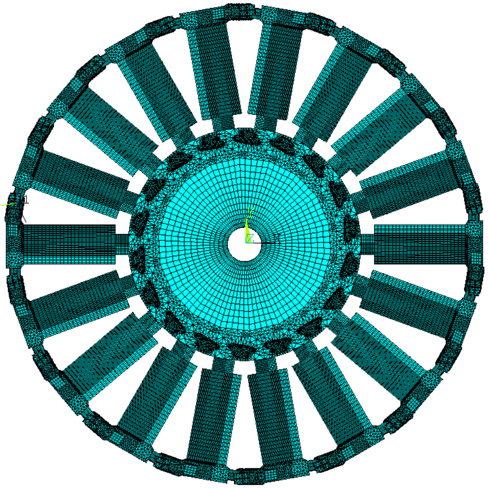


Different modal families exist, all of them characterized by the 0…$N/2$ ND sequence

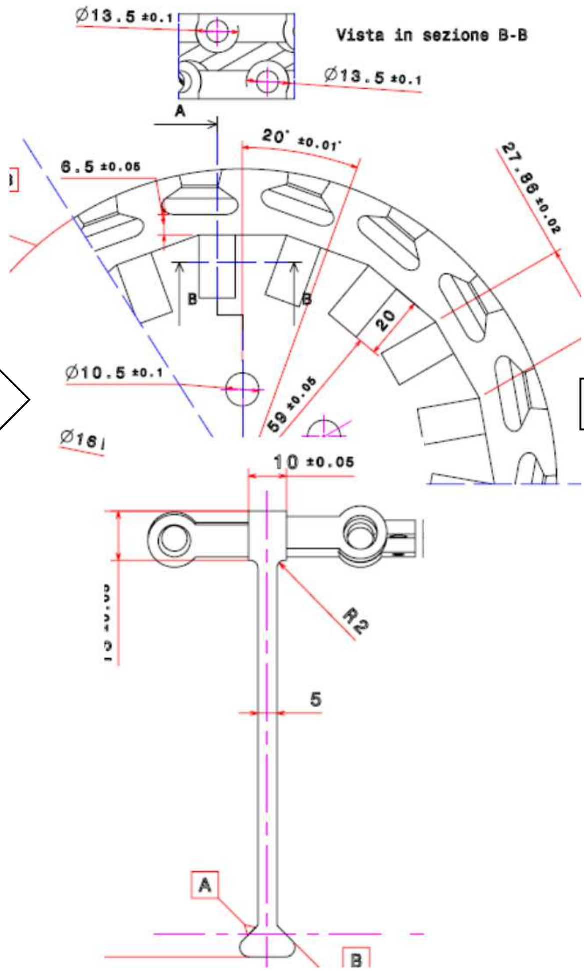


The effect of asymmetries in bladed disks

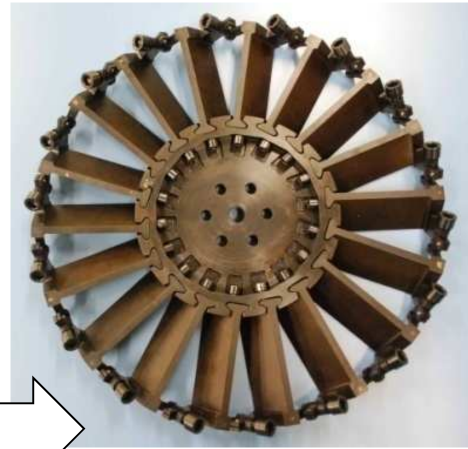
From FEM to reality

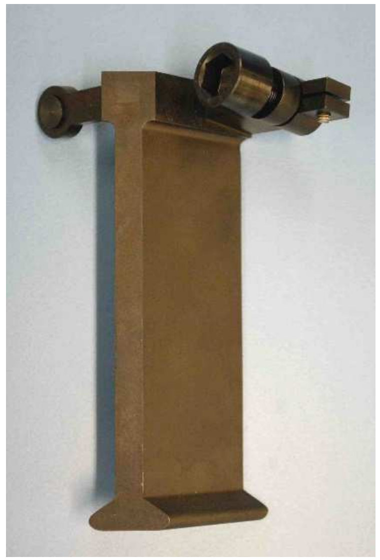


Technical Drawings



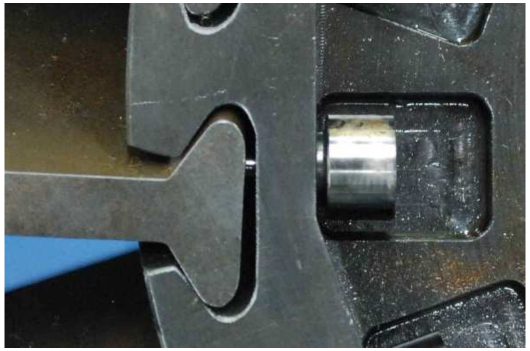
Manufacturing





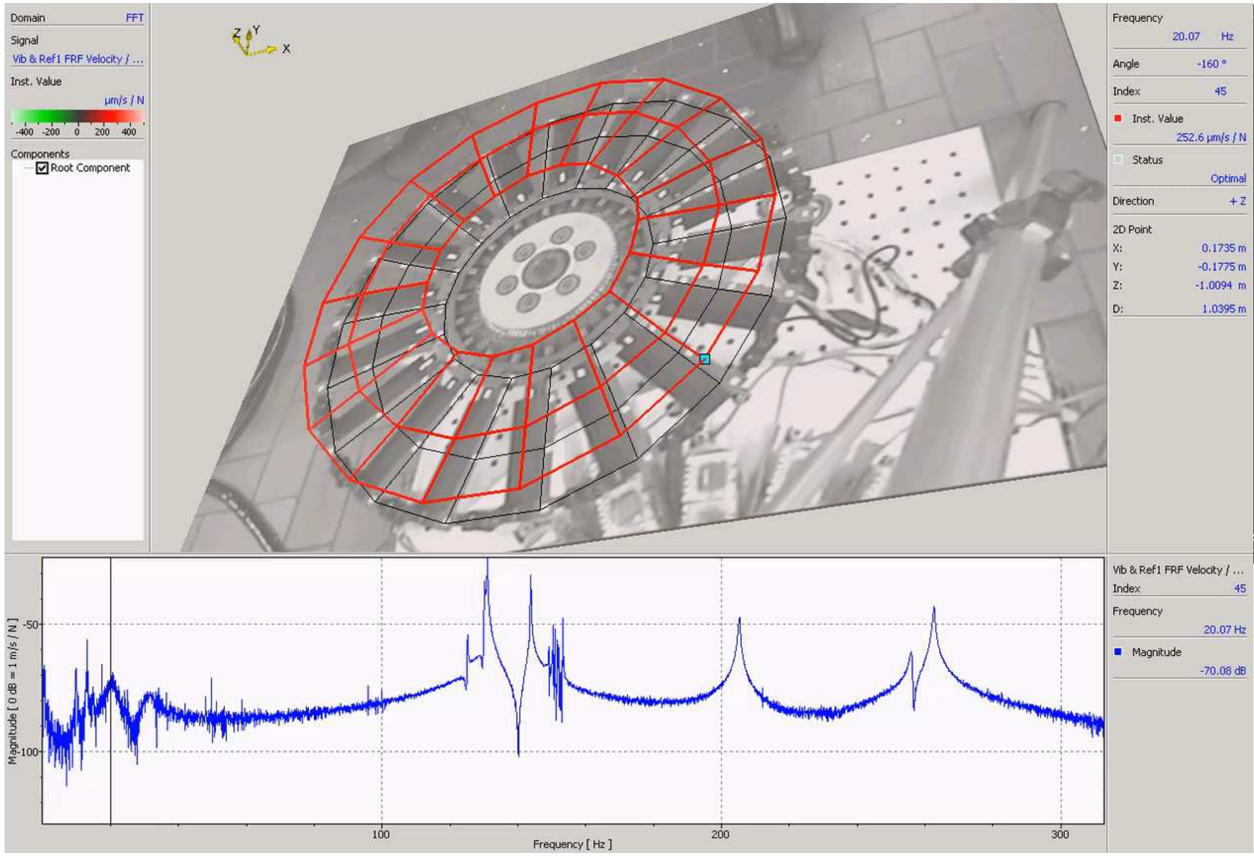
Experimental campaign

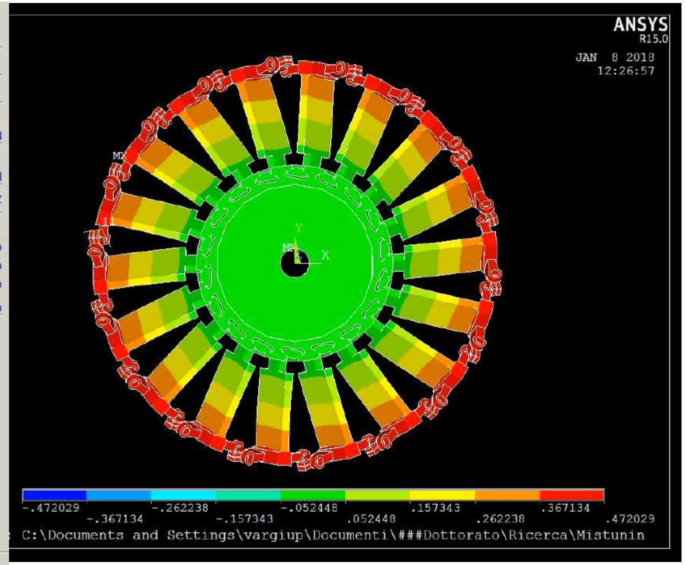
Testing: hammer test



Experimental campaign

Results and comparison with simulations



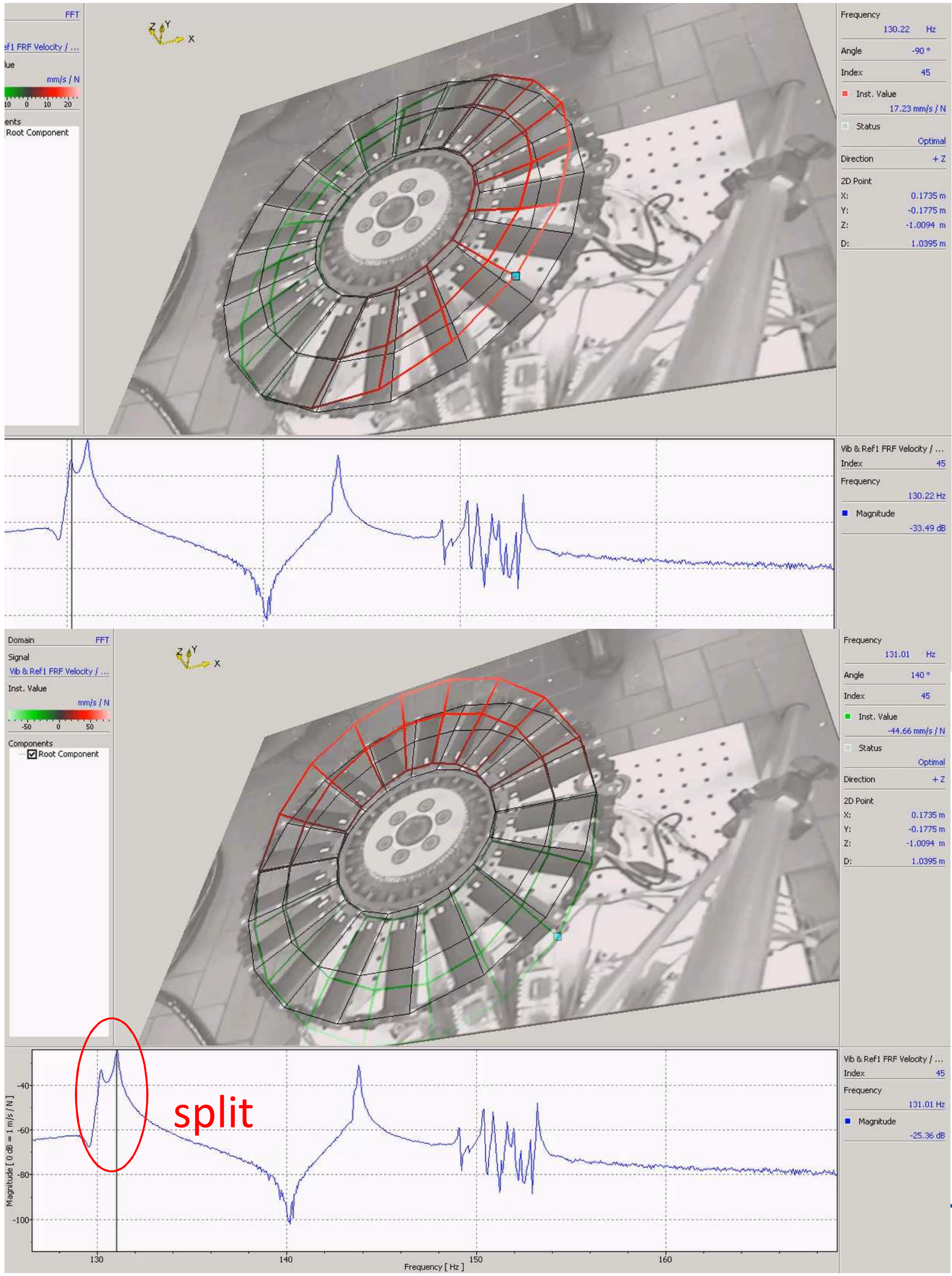


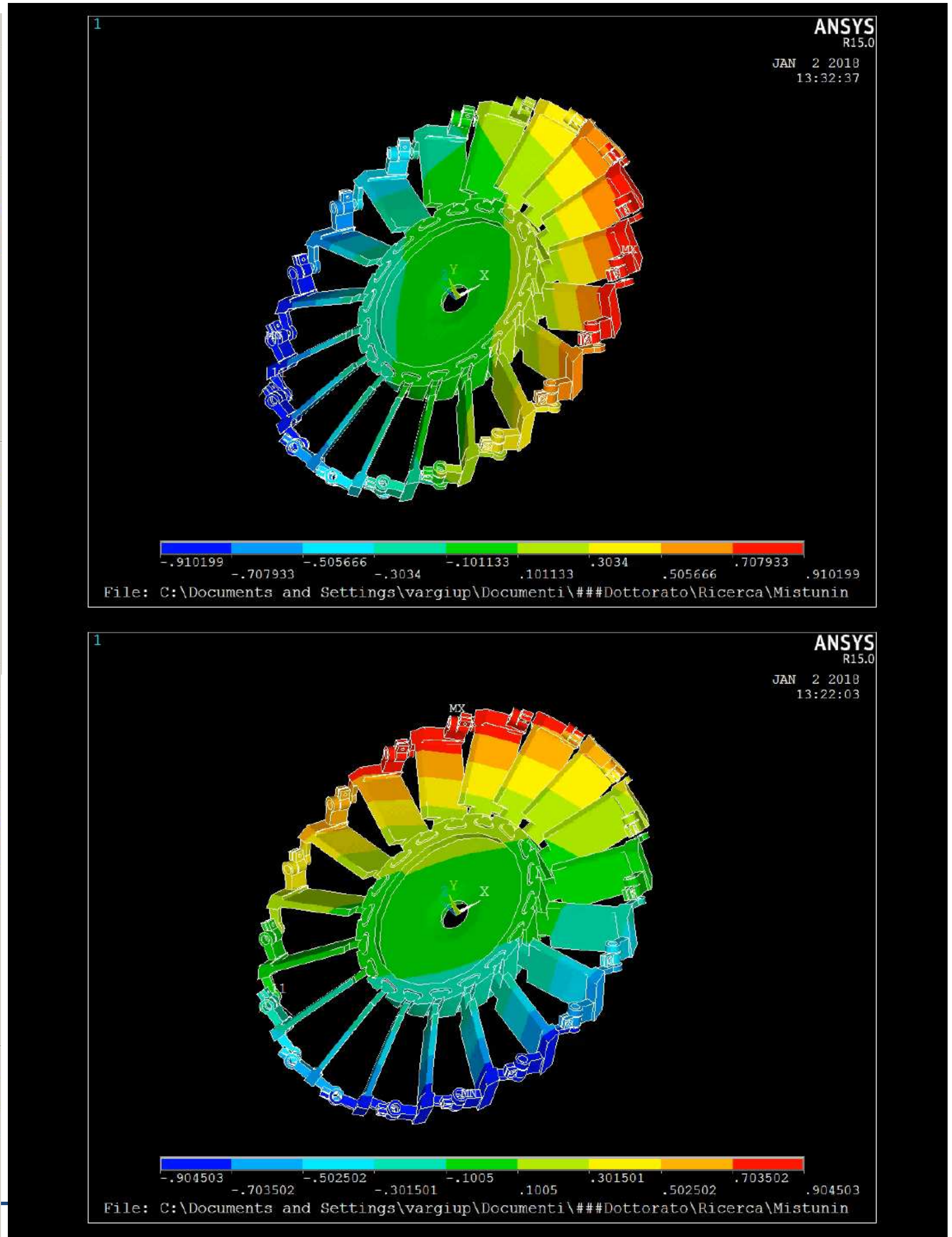
ND=0

fam 1 – 1F

fam 2 – 1F ortho …

The effect of asymmetries in bladed disks





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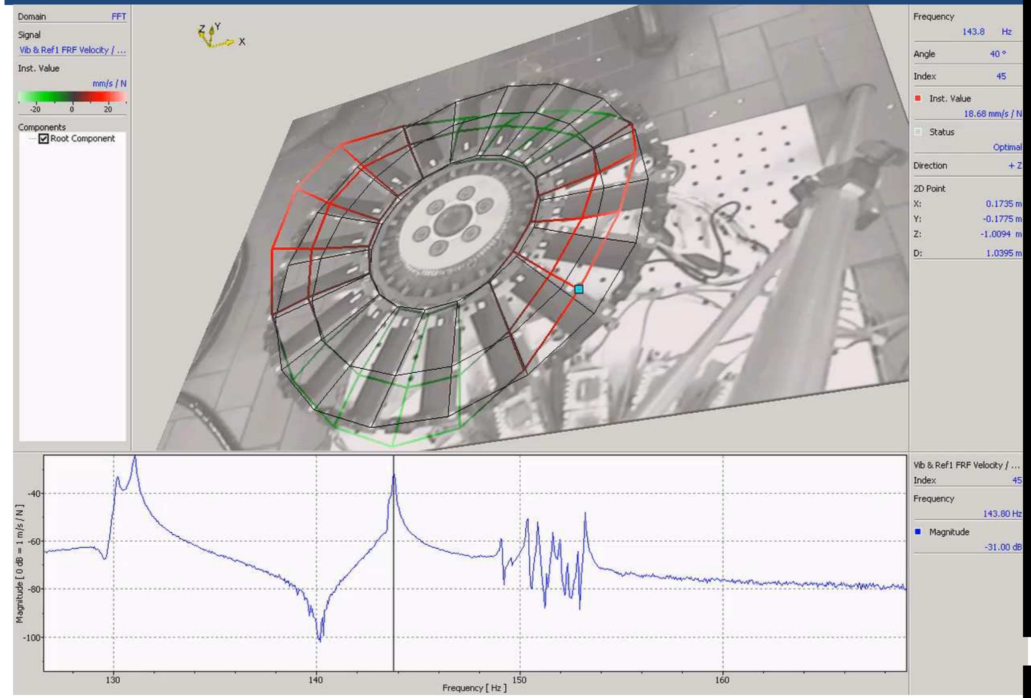
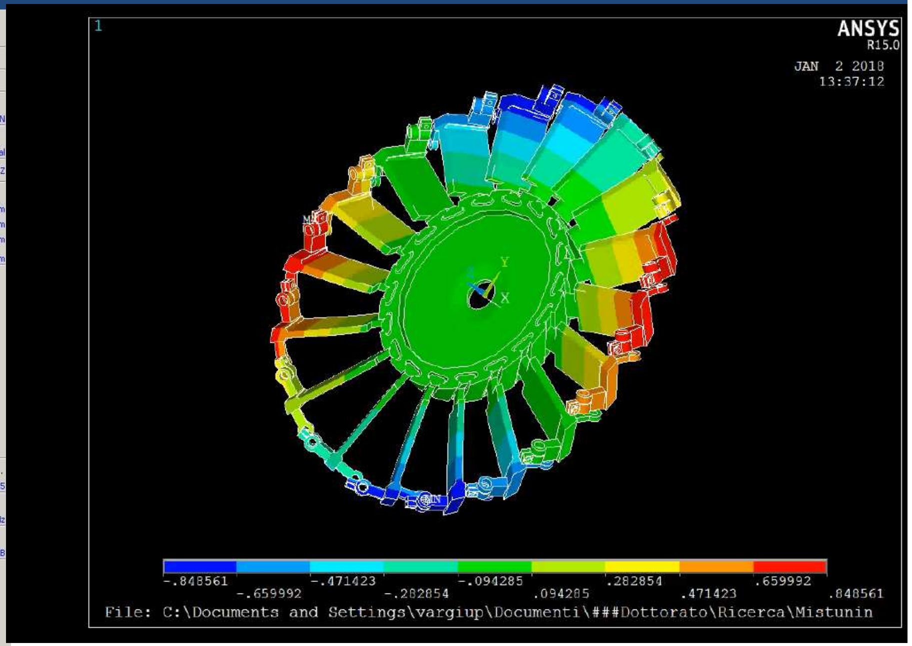
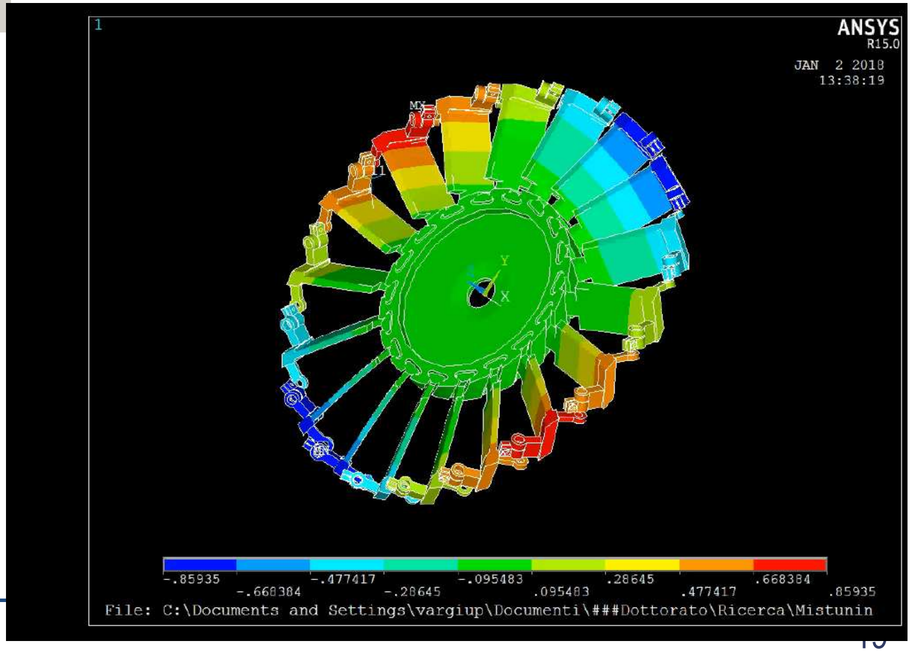
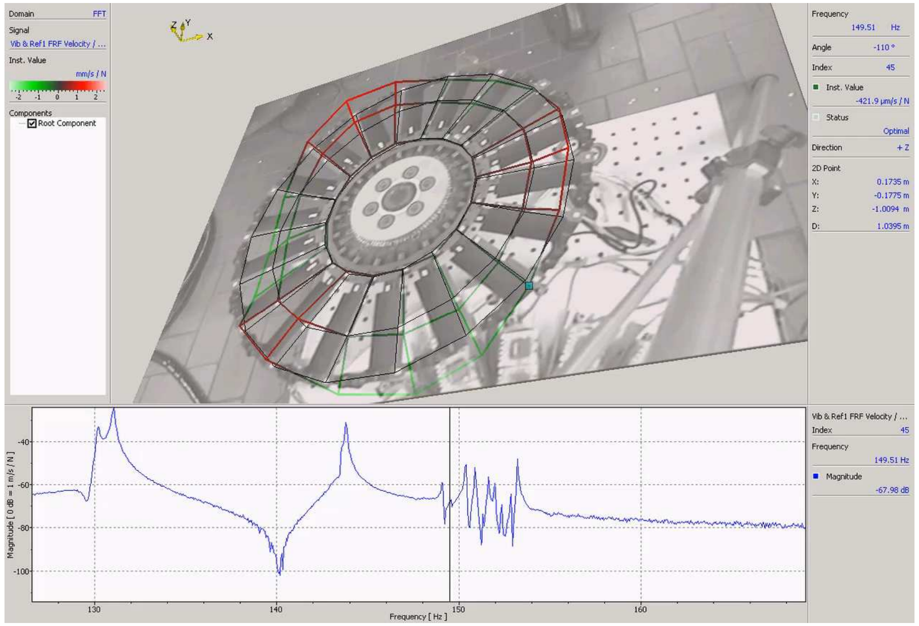


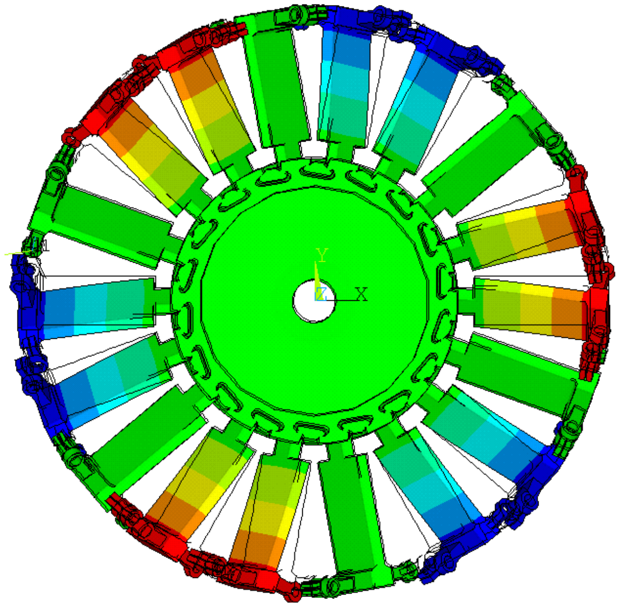


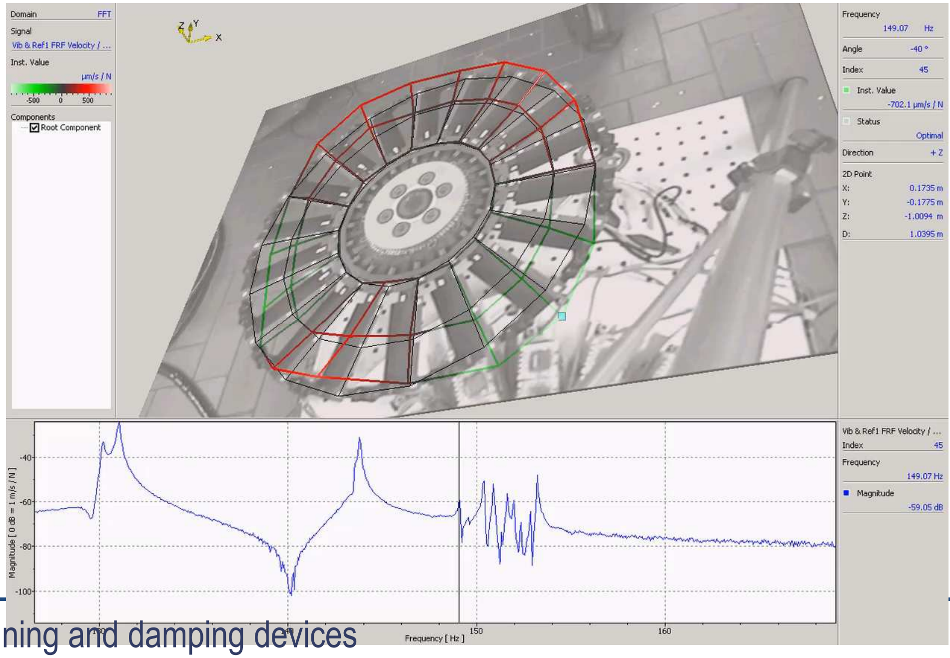


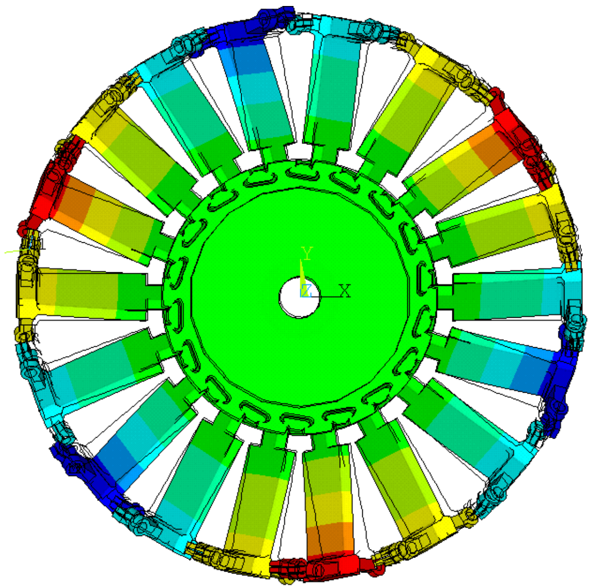
No show!

The effect of asymmetries in bladed disks

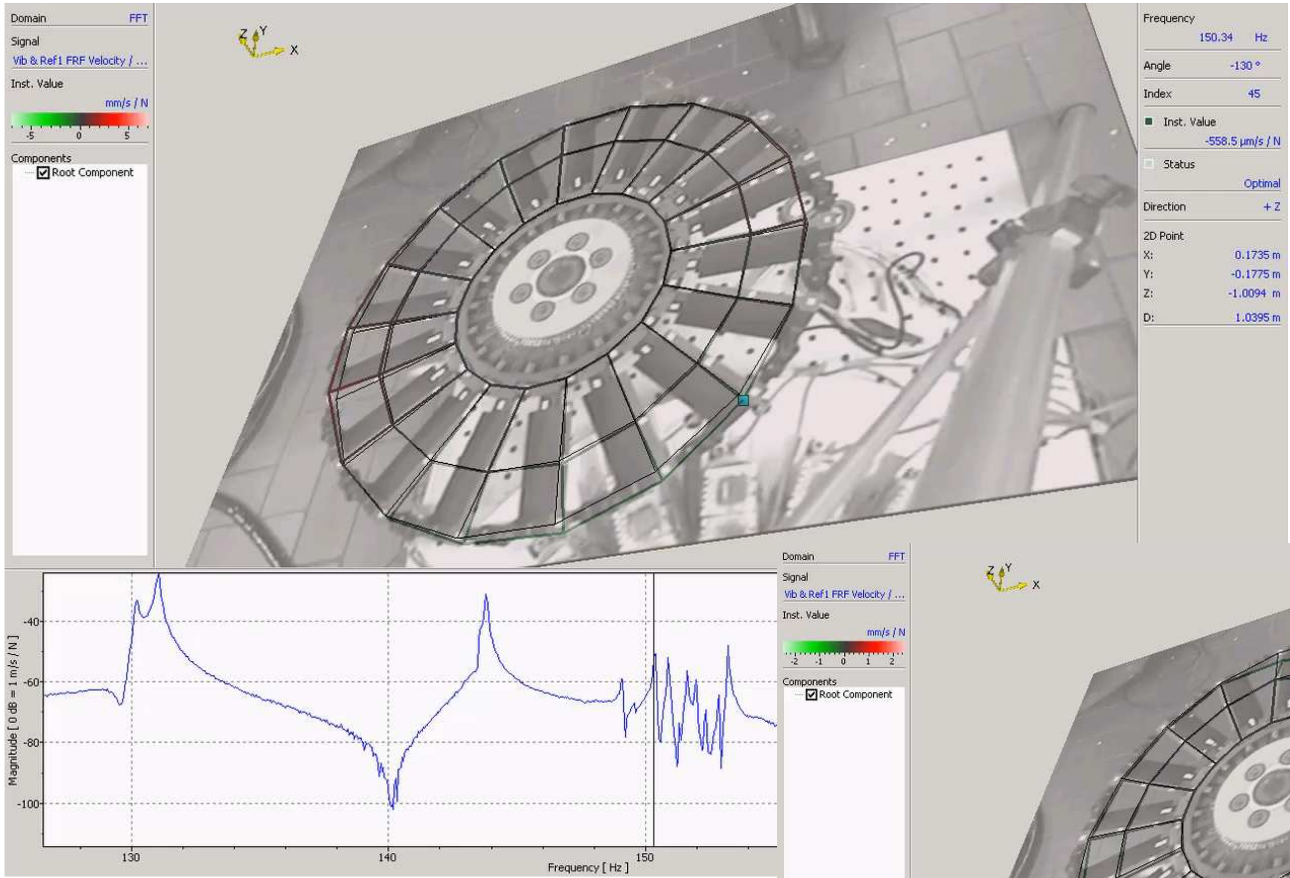


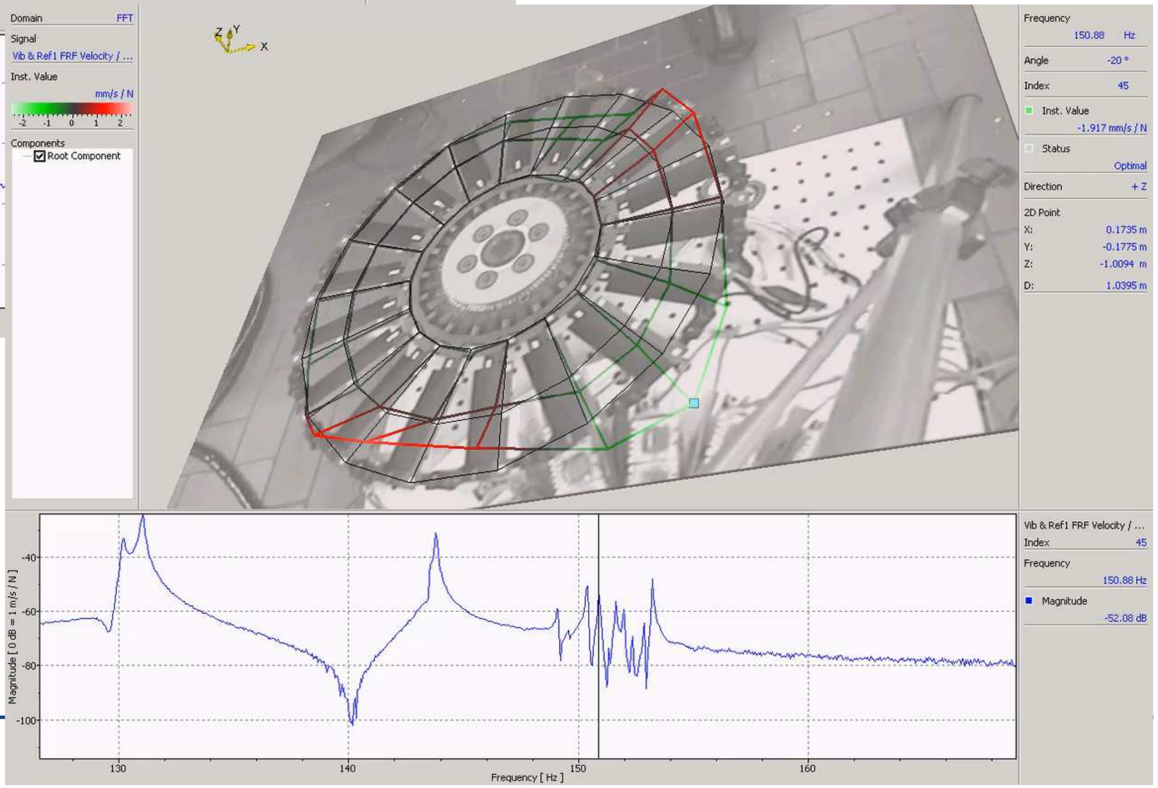






The effect of asymmetries in bladed disks





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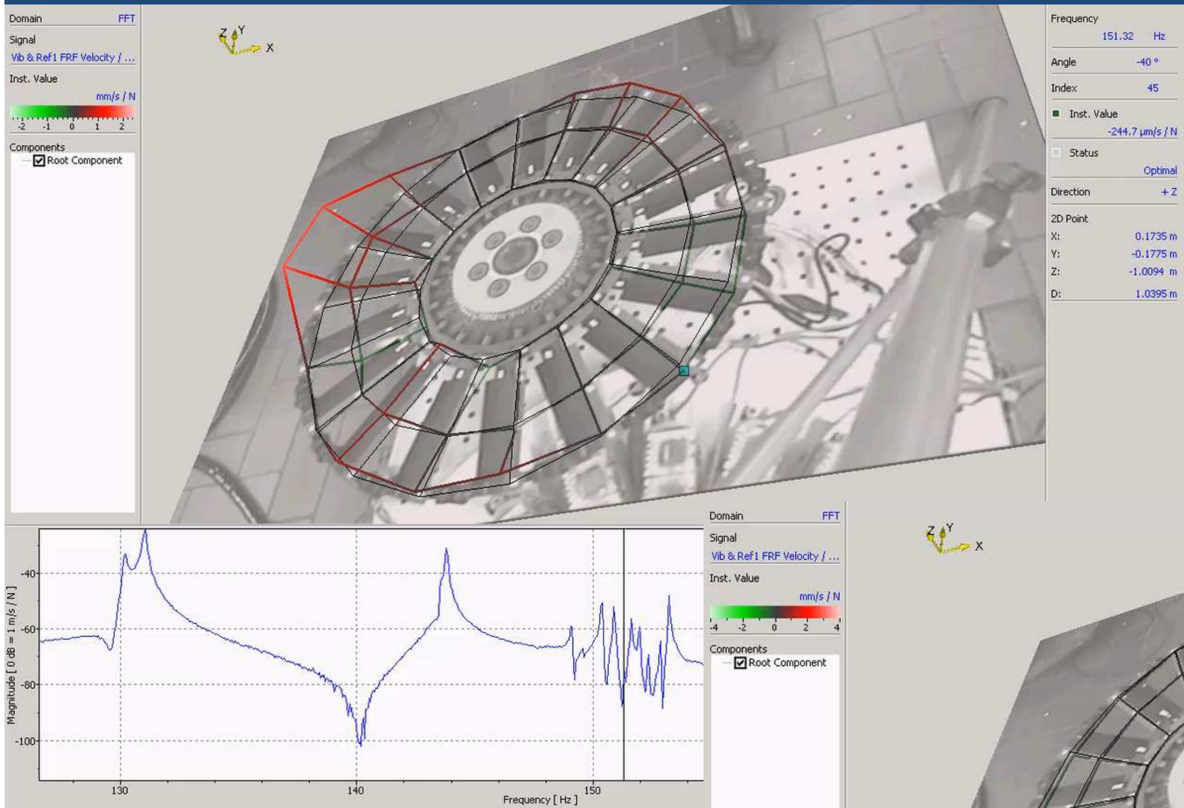
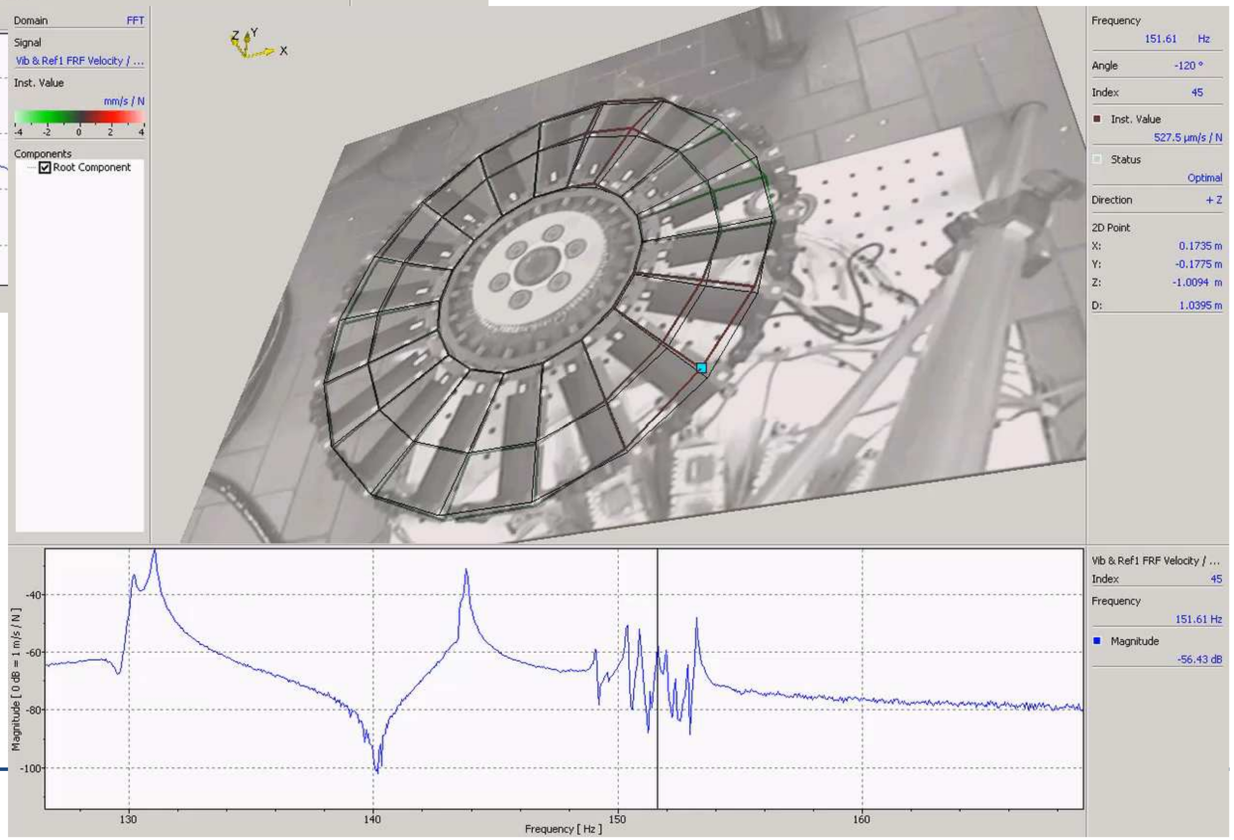


Response amplification for some blades



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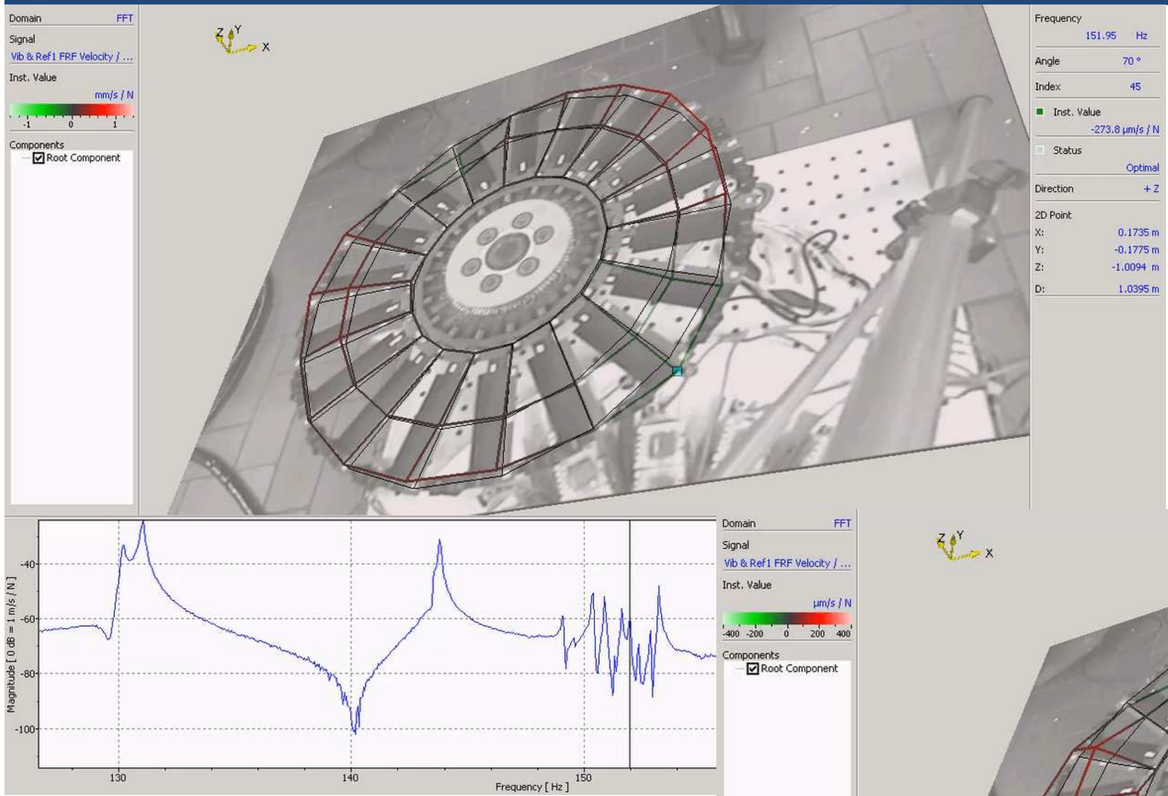

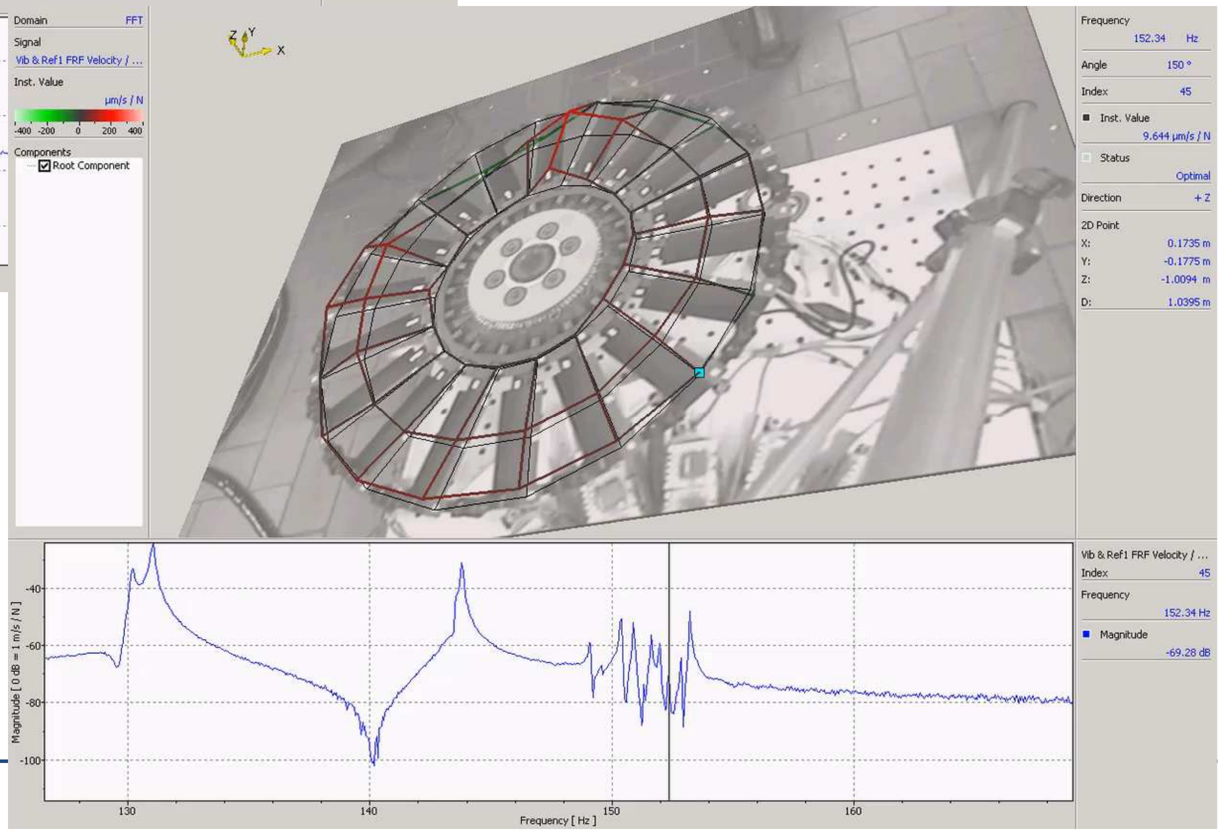

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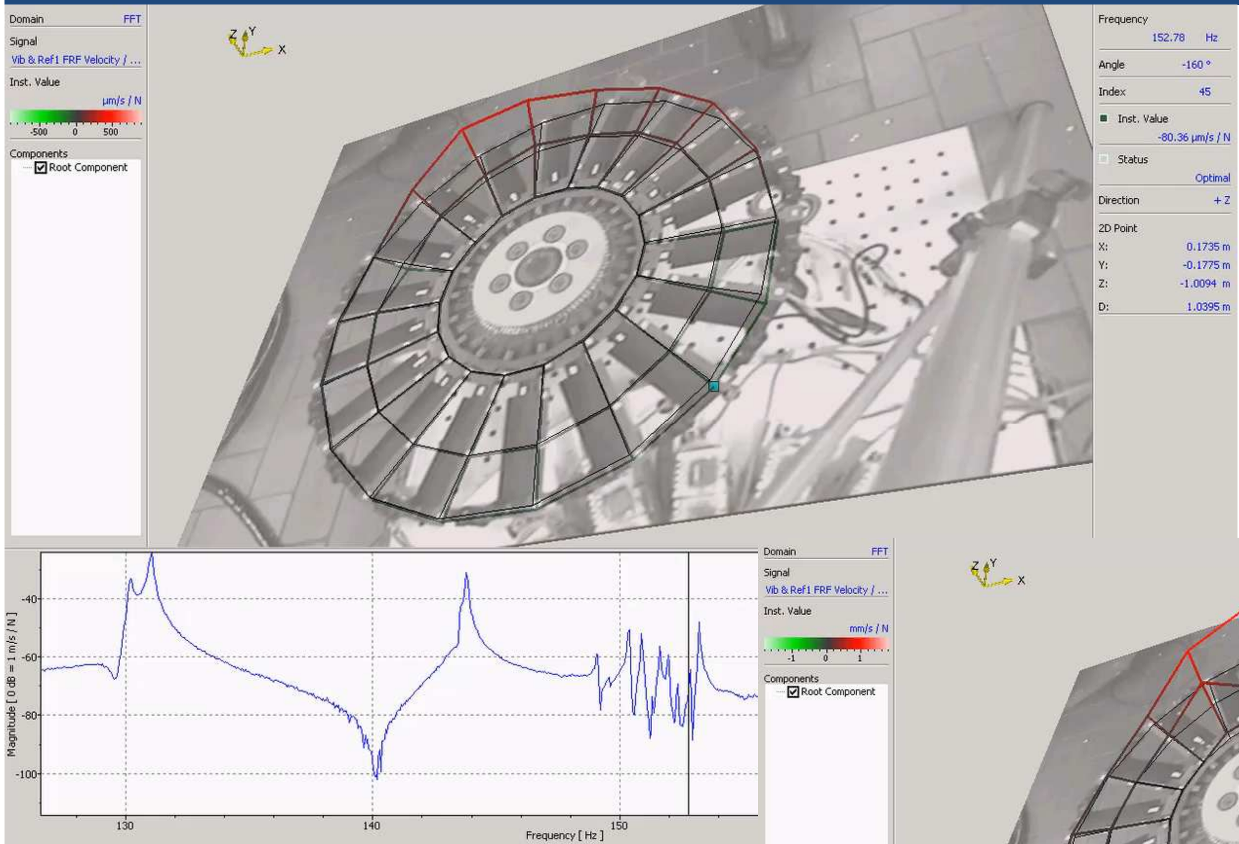
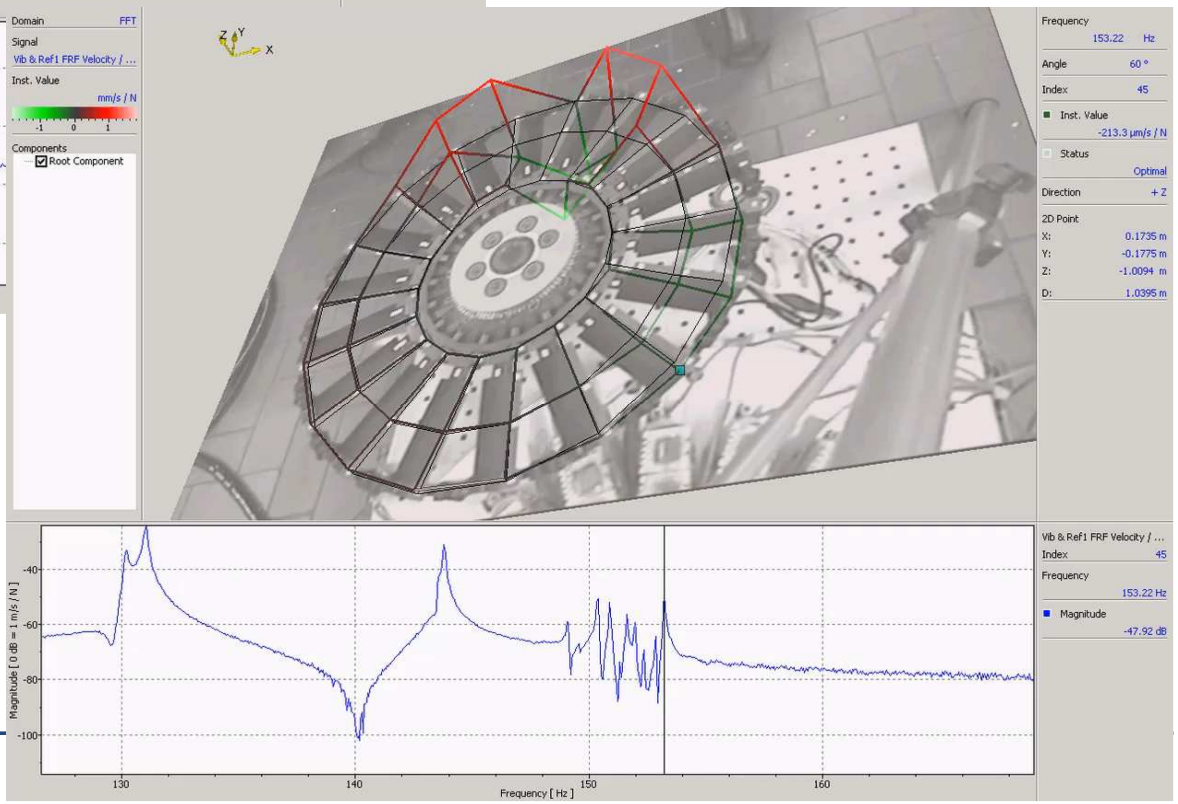
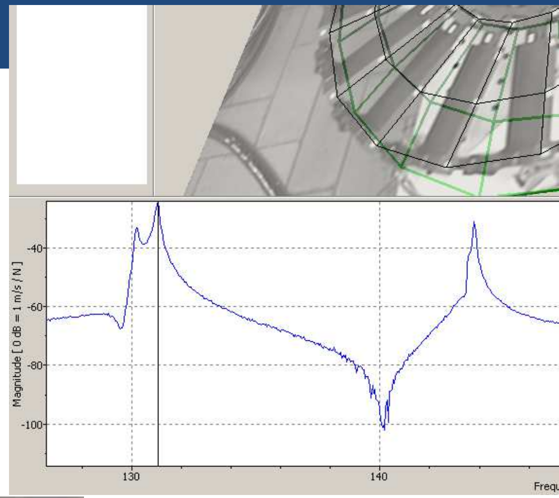


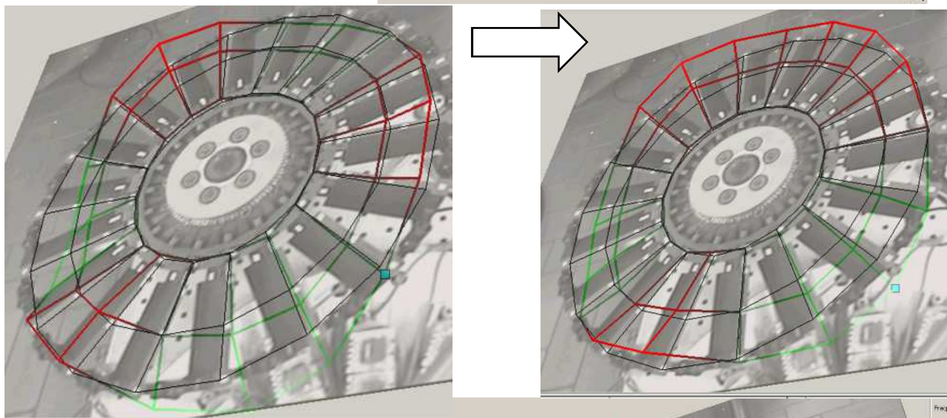


Response amplification for some blades

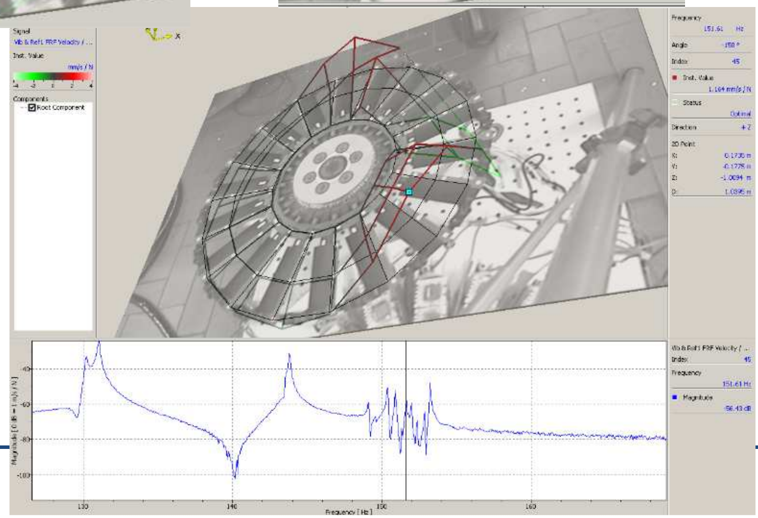
Symptoms of mistuning and causes



Split of the repeated frequency

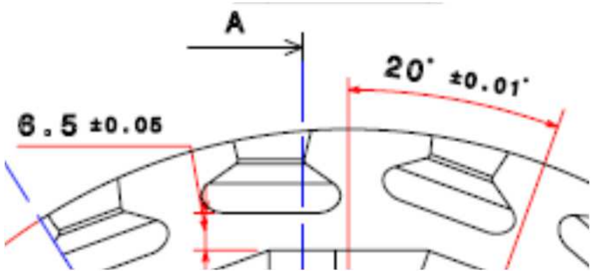


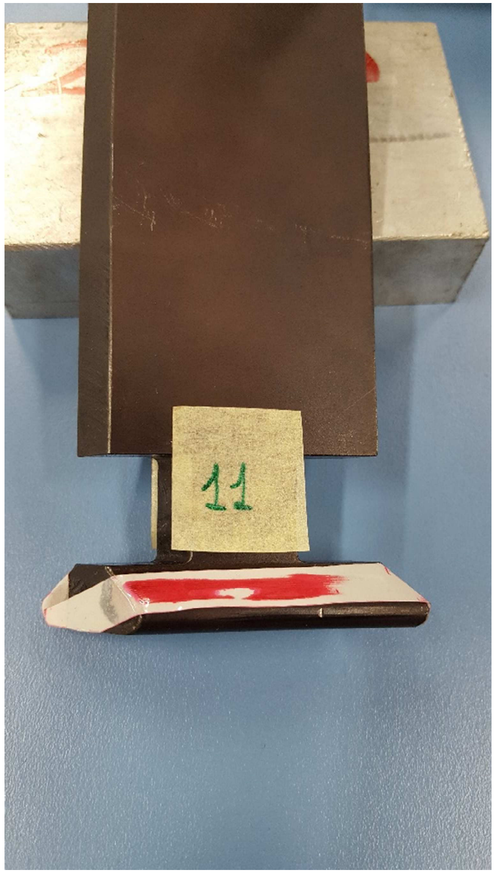
Nodal Diameter distortion (isolated mode shapes)

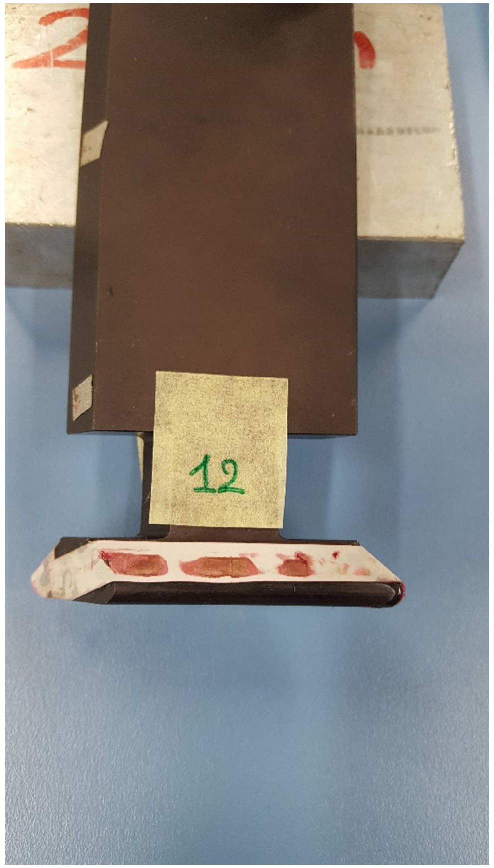


Amplification factor (high modal density regions)

Manufacturing tolerances

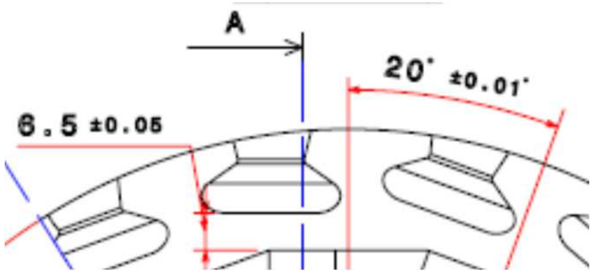




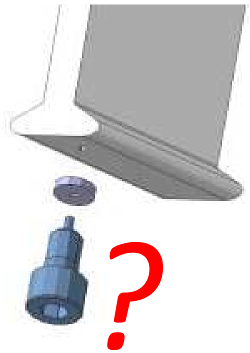


Symptoms of mistuning and causes

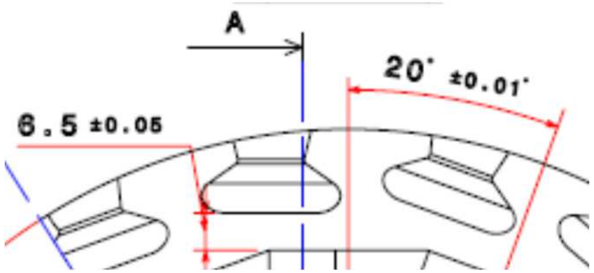
Manufacturing tolerances



Assembly

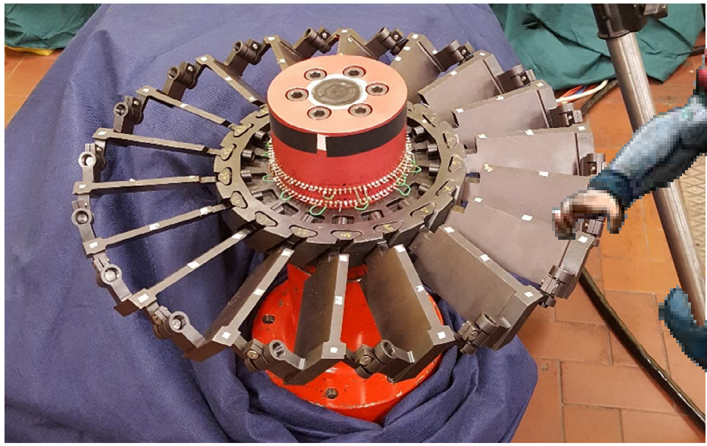


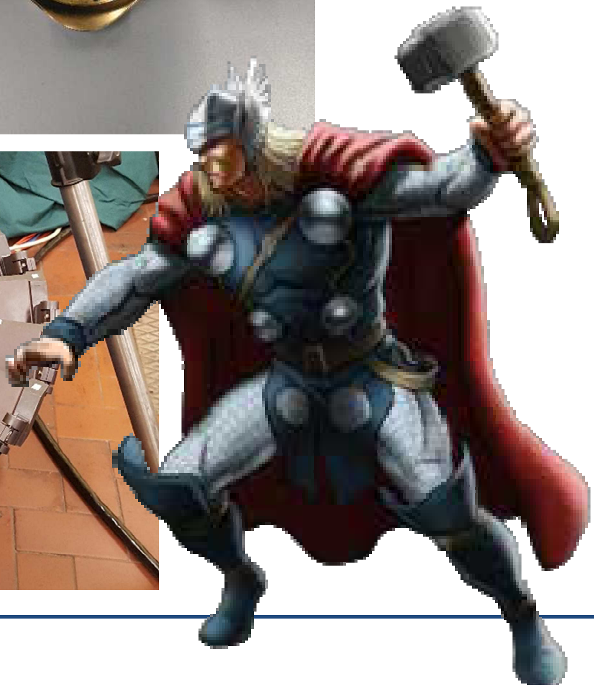
Manufacturing tolerances



Assembly

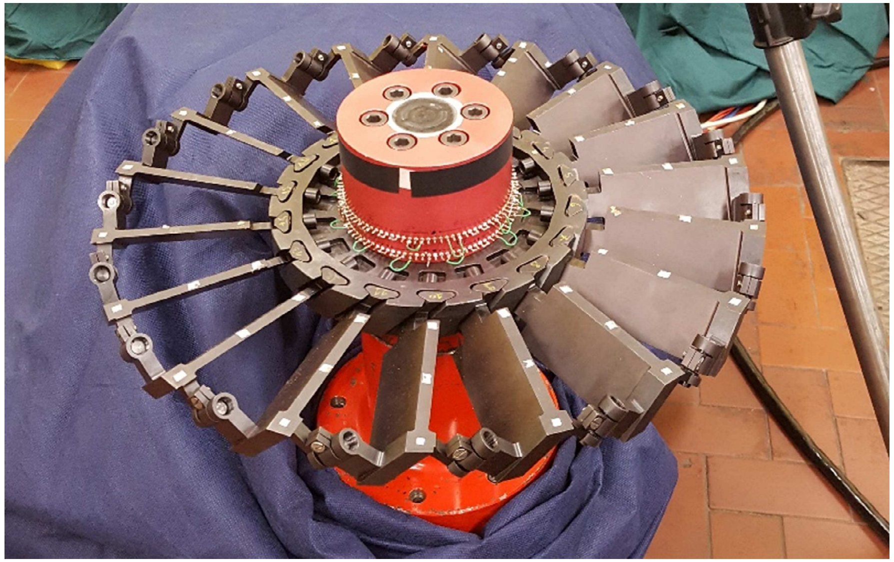
Wear in service

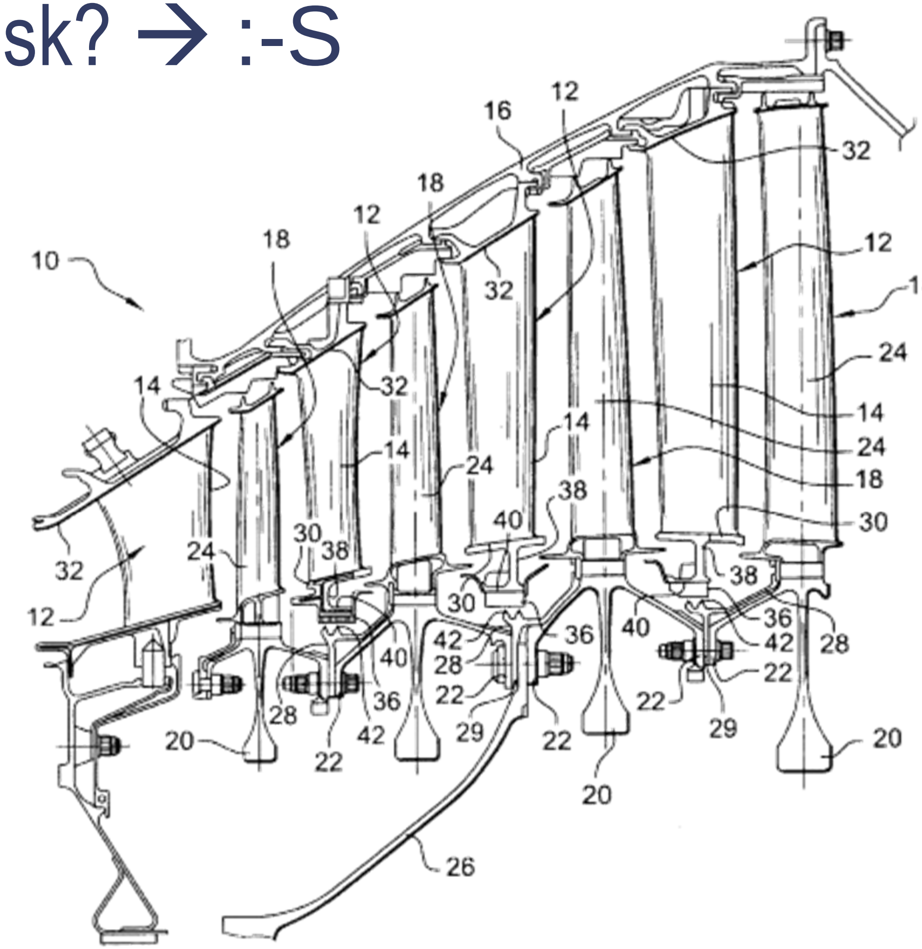




Symptoms of mistuning and causes

What if in a real bladed disk? → :-S

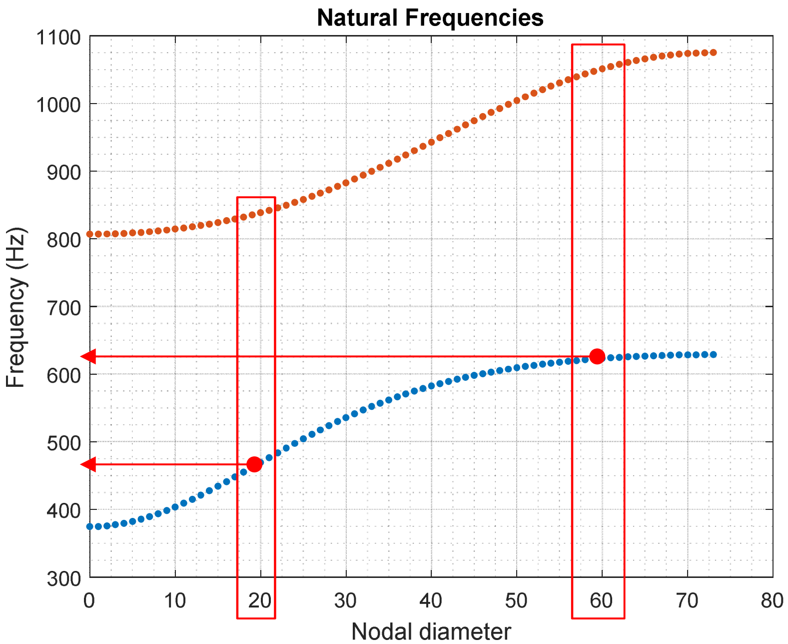




Фиг.1

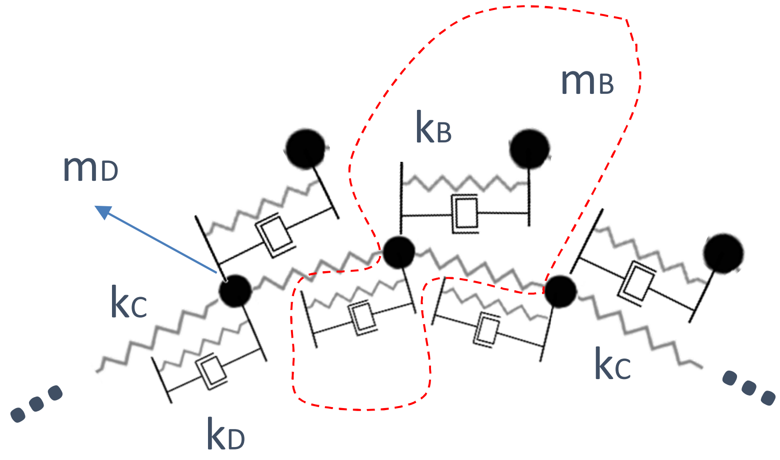
http://russianpatents.com/patent/253/2532868.html

Lumped parameters model



Isolated mode

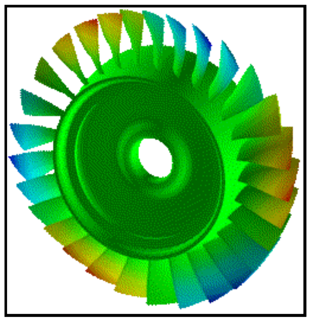
High modal density region



$N = 146\ sectors$

Excitation: travelling force with a given waveform along the hoop direction.
The number of waves is called Engine Order (EO).

$$EO = mN \pm ND \qquad m = 0 \rightarrow EO = ND = 20, 60$$

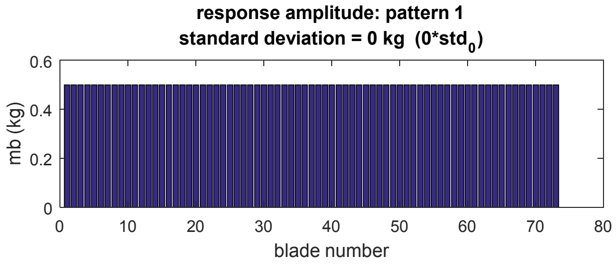


McGill University

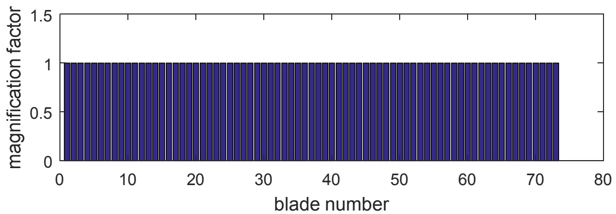
Lumped parameters model

Isolated mode, first family $EO = ND = 20$

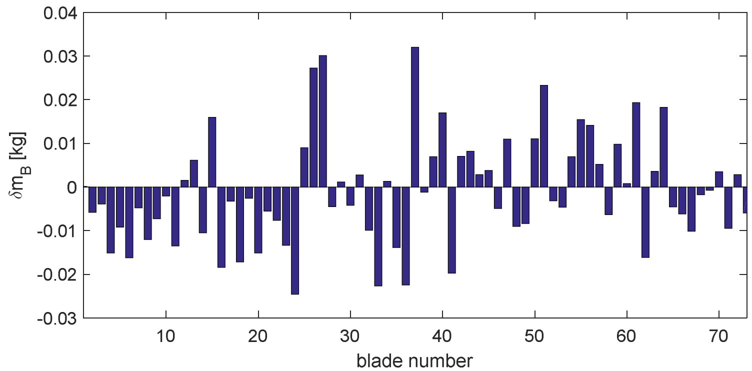
Tuned response, first 73 blades plotted

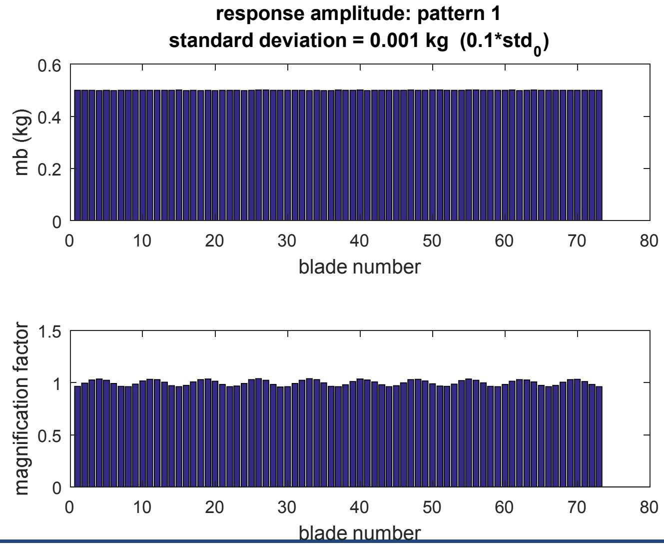


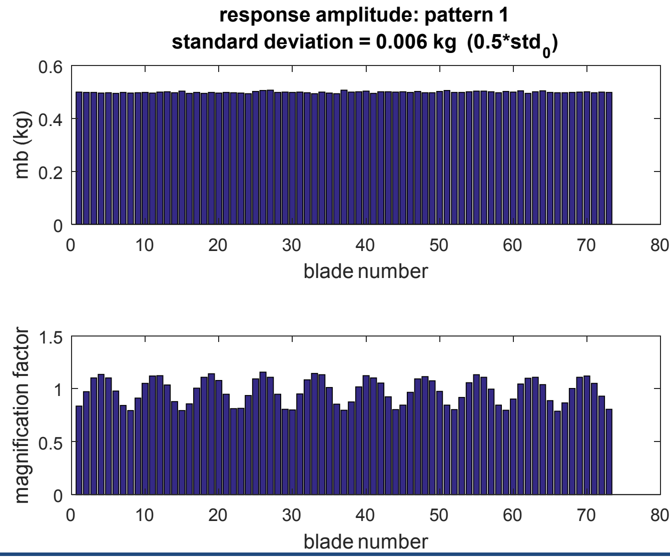
Tuned



A mistuning pattern is added on m_B with 0 mean value and standard deviation of 1.3% of the tuned blade mass (small mistuning)



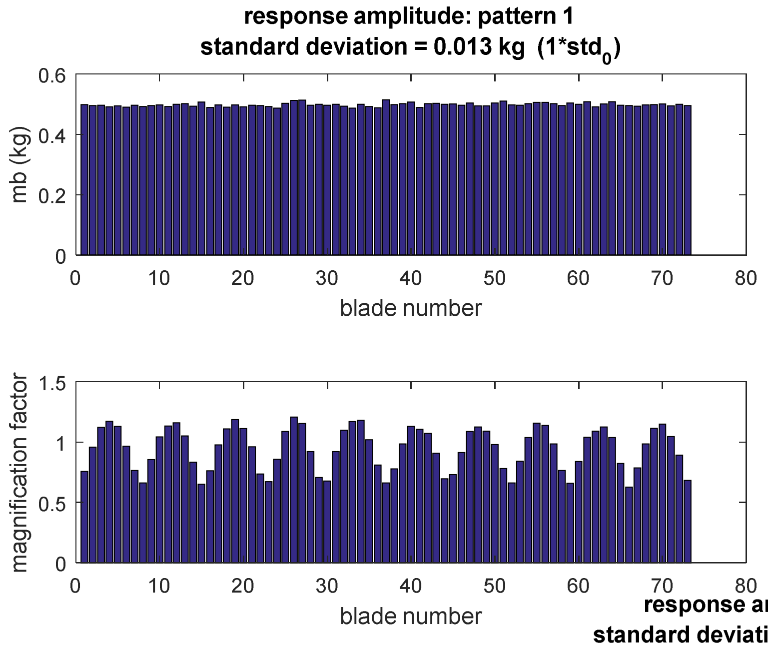


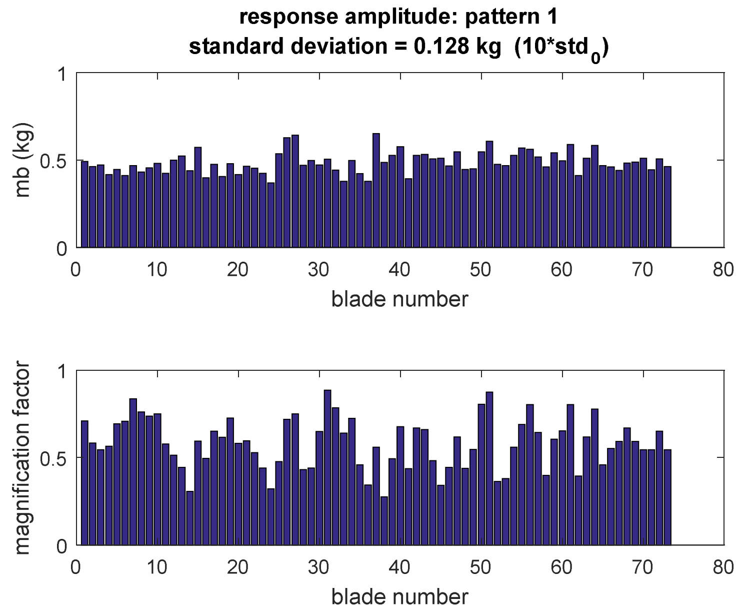


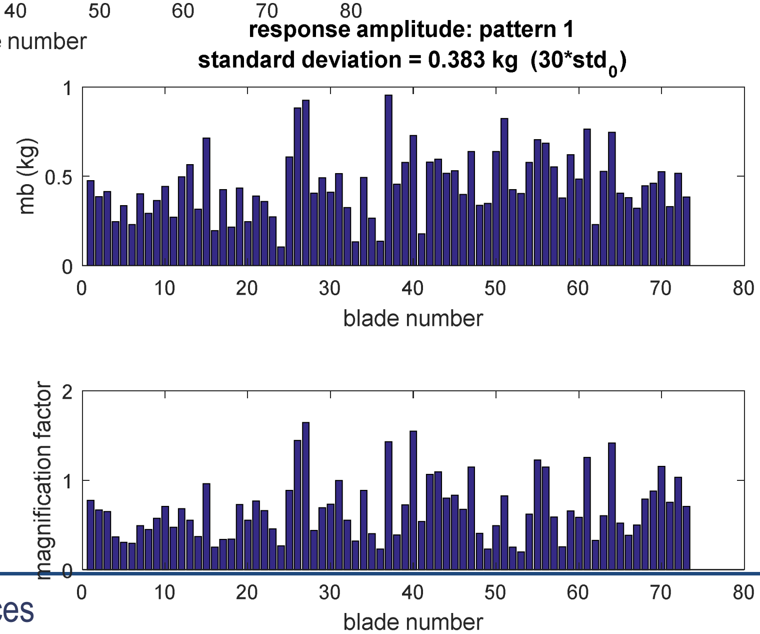
Lumped parameters model

Isolated mode, first family

$$EO = ND = 20$$







Large mistuning!

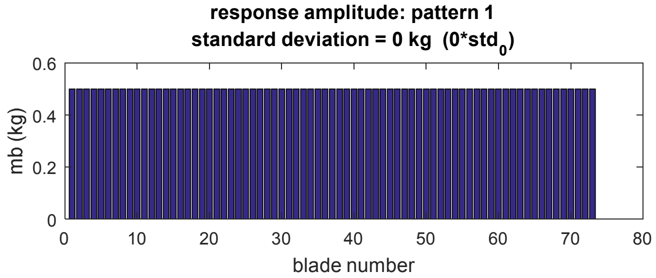
http://avherald.com/h?article=42290a67/0000

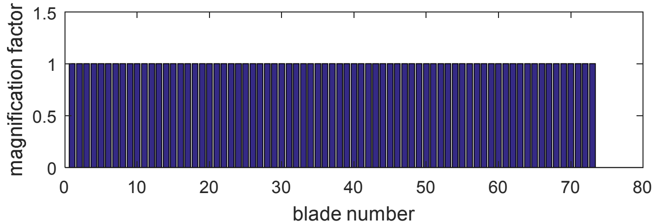
Lumped parameters model

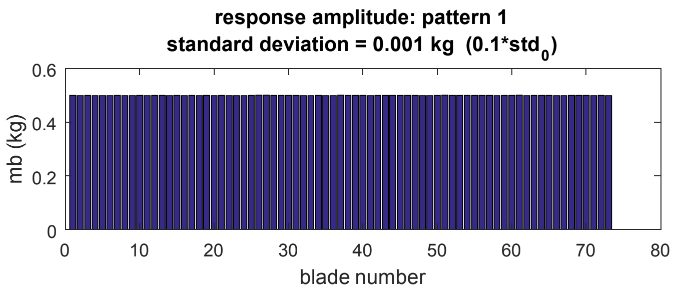
High modal density region, first family $EO = ND = 60$

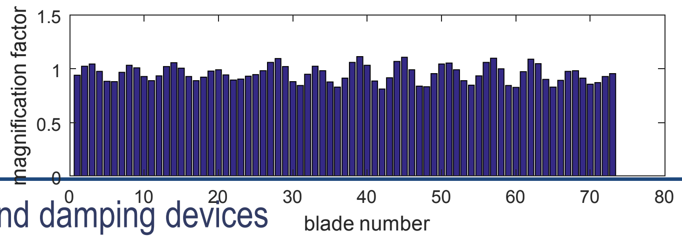
Tuned response, first 73 blades plotted

Tuned

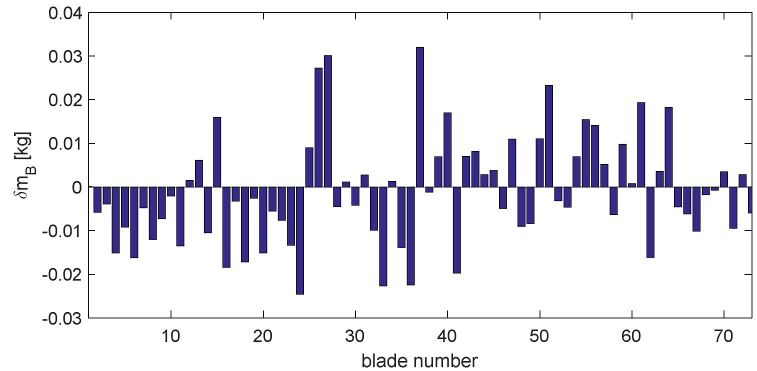


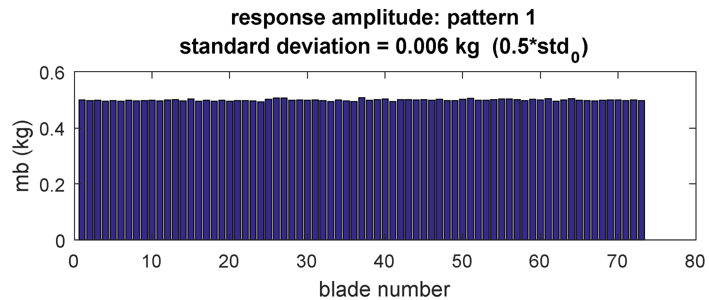


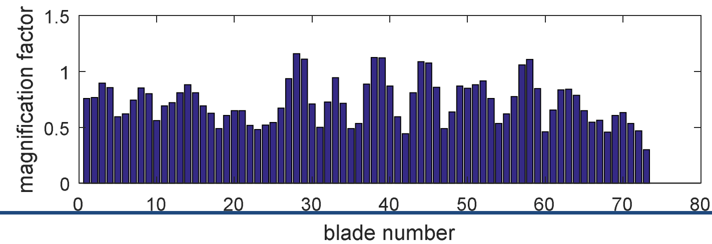




A mistuning pattern is added on m_B with 0 mean value and standard deviation of 1.3% of the tuned blade mass (small mistuning)



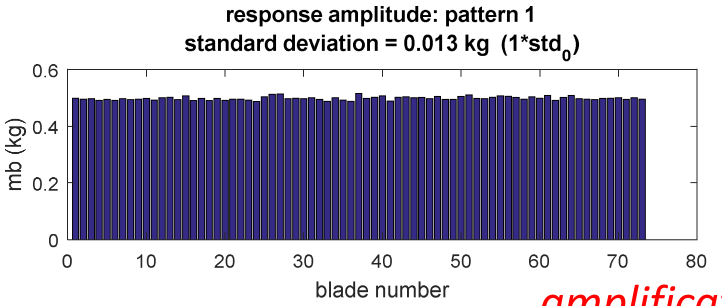




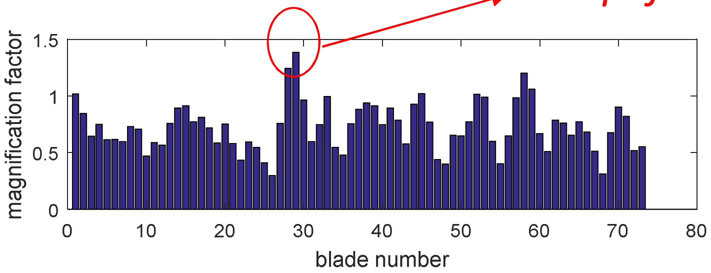
Lumped parameters model

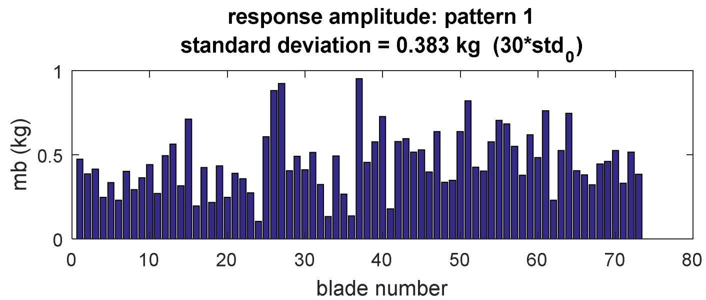
High modal density region, first family

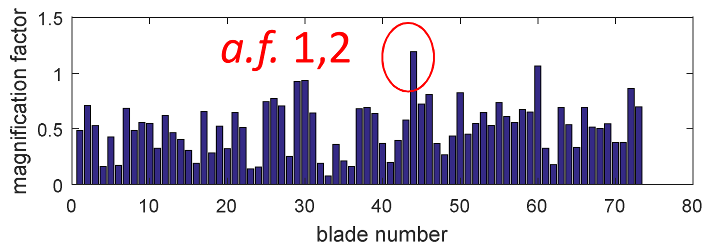
$EO = ND = 60$

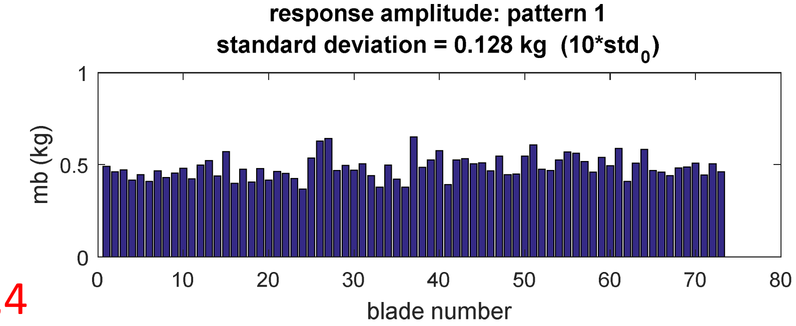


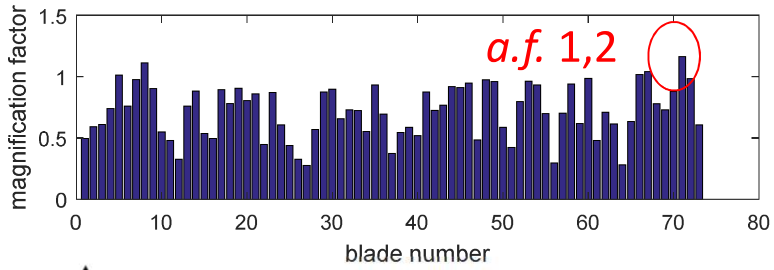
amplification factor: 1,4

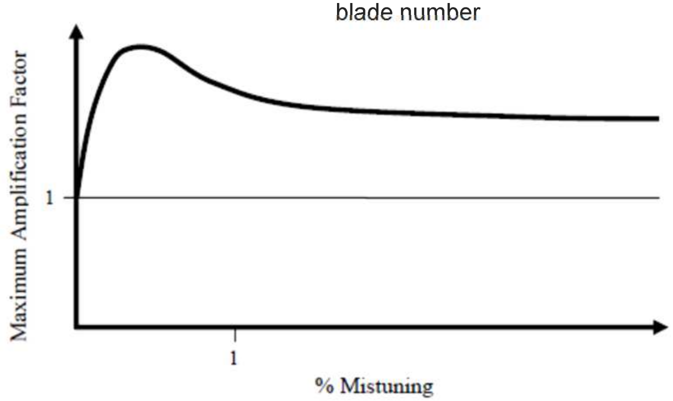












Whitehead *amplification factor* (1966):

$$\max \quad a.f. = \frac{1+\sqrt{N}}{2}$$

Numerical techniques to take into account mistuning: a literature overview

Mistuning models

- Carnegie Mellon University (USA) – Griffin
- University of Michigan (USA) – Castanier, Pierre, Epureanu
- Imperial College (UK) – Ewins, Petrov
- Brandenburg University of Technology Cottbuss – Kühhorn, Beirov

Reduction Techniques

- Carnegie Mellon University (USA) – Griffin
- University of Michigan (USA) – Castanier, Pierre
- Imperial College (UK) – Ewins, Petrov
- Politecnico di Torino (ITA) – Vargiu
- Université Paris-Est (FR) – Soize

Mistuning Identification

- Carnegie Mellon University (USA) – Griffin
- Ecole Centrale de Lyon (FR) – Thouverez
- Brandenburg University of Technology Cottbuss – Kühhorn, Beirov

Numerical techniques to take into account mistuning: a literature overview

Mistuning models

MISTUNING

GEOMETRICAL MISTUNING

FREQUENCY MISTUNING

Machining, assembly, wear, material

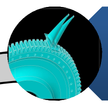
EFFECT

Modification of the natural frequency values of the system wrt the tuned frequency

Usually, mistuning is considered in terms of frequency mistuning.

***BLADE* FREQUENCY MISTUNING**

Most used, mistuning is associated to the blade differences



***SECTOR* FREQUENCY MISTUNING**

Least used, mistuning is associated to blade-disk interface and more in general disk sector asymmetries

Numerical techniques to take into account mistuning: a literature overview

Reduction techniques: useful during design

- Random mistuning pattern;
- If small, mistuning may produce higher amplificaton factors.

- Statistic approach: Monte Carlo technique

- If full FE models are used, the computational cost is prohibitive

- REDUCTION TECHNIQUES

Main requirements:

- The tuned (reference) highly detailed FE model must be modelled using only one basic sector to be treated with cyclic symetry hypothesis
- The Reduced Order Model (ROM) must allow:
- the introduction of mistuning.
- The calculation of the forced response for any EO excitation.

FREQUENCY MISTUNING $$\delta_i = \frac{\omega_i - \omega_0}{\omega_0} \%$$

ω_i = “mistuned” frequency; ω_0 = “tuned” frequency

Numerical techniques to take into account mistuning: a literature overview

Reduction techniques: useful during design

$$\underbrace{([K]}_{N \times N}-\omega^2\underbrace{[M]}_{N \times N}+i\omega\underbrace{[C]}_{N \times N})\cdot\{x\}=\{0\}; \xrightarrow{n \ll N} (\underbrace{[\Psi]^T[K][\Psi]}_{\substack{[k] \\ n \times n}}-\omega^2\underbrace{[\Psi]^T[M][\Psi]}_{\substack{[m] \\ n \times n}}+i\omega\underbrace{[\Psi]^T[C][\Psi]}_{\substack{[c] \\ n \times n}})\cdot\{q\}=\{0\};$$

$$\underbrace{\{x\}}_{N \times n}=\underbrace{[\Psi]}_{N \times n}\cdot\underbrace{\{q\}}_{n \times 1};$$

Component Mode Sinthesis (C.M.S.)

Example: Craig-Bampton (1968).

Bladed disk divided into its main components: Disk and blades

$\{q\} \Rightarrow$

- ✓ Modal coordinates (slave modes);
- ✓ Physical dofs (master nodes).

Features: - Tuned system representation improves with the number of dofs; - Frequency mistuning.

Mistuning: Large number of master dofs

REDUCE, C.M.M.

Generalized, Normal Mode Reduction

Bladed disk studied as one body

$\{q\} \Rightarrow$

- ✓ modal coordinates;

Features: - Very good accuracy of the *tuned system*; - Geometrical mistuning.

Mistuning: Product of matrices with big size.

S.N.M., F.M.M., MIST.RES.

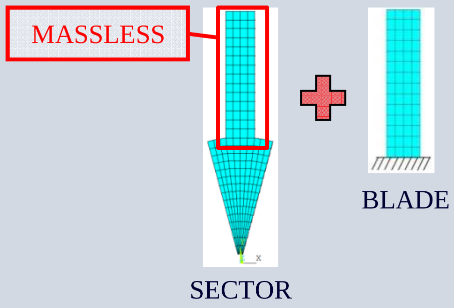
Numerical techniques to take into account mistuning: a literature overview

Reduction techniques: useful during design

REDUCE

Pierre,1997

Idea Blade displacements are a linear combination of two contributions: cantilever (mistuned) blade motion and motion driven by the disk.



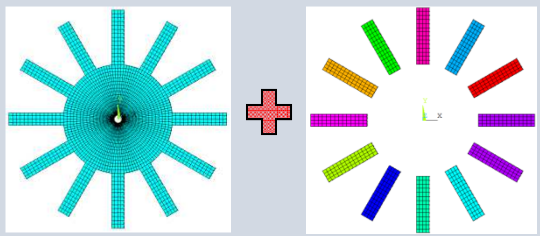
$[\Psi] \Rightarrow [\Phi^{SECTOR}], [\Phi^{BLADE}]$

$\{q\} \Rightarrow$ Modal coordinates

C.M.M. (Component Mode Mistuning)

Pierre, 2007

Idea Matrix perturbation of the tuned condition = separate component with an interface with the tuned bladed disk.



$[M^o], [K^o]$ $[\Delta M], [\Delta K]$

$[\Psi] \Rightarrow [\Phi^{TUNED}]$

$\{q\} \Rightarrow$ Modal coordinates

I.M.M. (Integral Mistunign Model) Vargiu, 2012

N.E.W.T. (Nodal Energy Weight Transformation) Fitzner, 2014

Idea Mistuning can be associated to the disk sector and not only to the cantilever blade

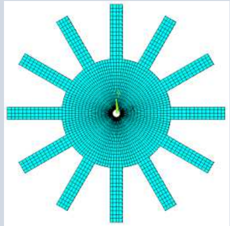
Numerical techniques to take into account mistuning: a literature overview

Reduction techniques: useful during design

S.N.M. (Subset of Nominal Modes)

Griffin, 2001

Idea Mistuned modal shapes = linear combination of tuned mode shapes within the same frequency range.



$[M^o]+[\Delta M],[K^o]+[\Delta K]$

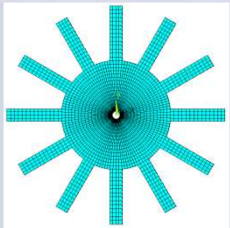
$[\Psi] \Rightarrow [\Phi^{TUNED}]$

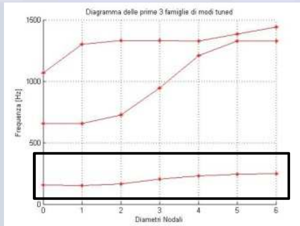
$\{q\} \Rightarrow$ Modal coordinates

F.M.M. (Fundamental Mistuning Model)

Griffin, 2002

Idea Application to a single modal family, isolate (no veering regions), flat.





$[M^o]+[\Delta M],[K^o]+[\Delta K]$

$[\Psi] \Rightarrow [\Phi^{TUNED}]$

$\{q\} \Rightarrow$ Modal coordinates

MIST.RES.

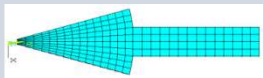
Ewins, 2002

Idea Sherman-Morrison-Woodbury formula to invert the dynamic stiffness matrix of the mistuned system.

$$[D(\omega)]^{-1} = \Big([D_0]+\underbrace{[U\cdot V^T]}_{[\Delta D]}\Big)^{-1} = [D_0]^{-1}-[D_0]^{-1}\cdot[U]\cdot\left([I]+[V]^T\cdot[D_0]^{-1}\cdot[U]\right)^{-1}\cdot[V]^T\cdot[D_0]^{-1}$$

$\{q\} \Rightarrow$ Physical dofs

$\{x\}= f(x_o, D_o^{-1}) \Rightarrow [\Phi^{TUNED}] \Rightarrow$



+ Cyclic symmetry

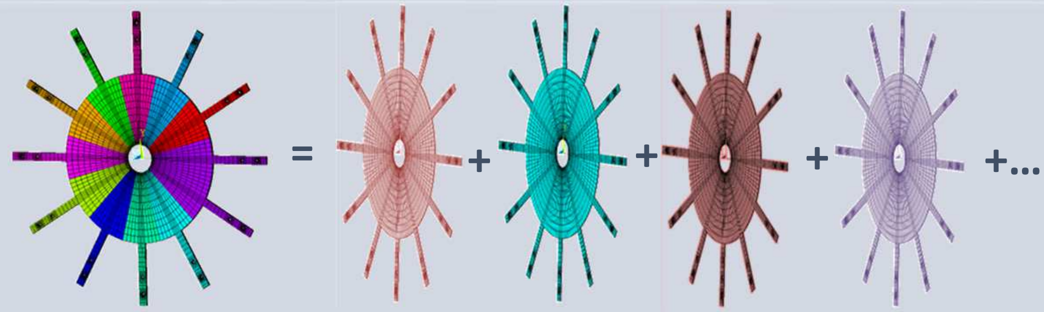
Numerical techniques to take into account mistuning: a literature overview

Reduction techniques: useful during design

LARGE MISTUNING (Subset of Normal Modes)

Mbaye, Soize, 2010

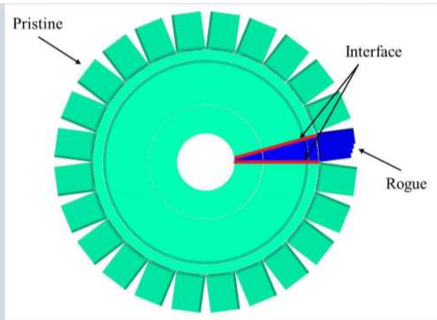
Idea Each mistuned sector is used to build a ‘tuned’ disk and a modal basis is obtained as a set of cyclically symmetric mode shapes.



P.R.I.M.E. (Large mistuning)

Madden, 2012

Idea Pristine-Rogue-Interface basic component, in this case the projection of the mistuned model is made using sector level mode shapes (low computational cost).



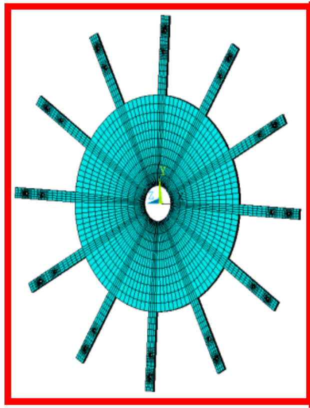
N-PRIME (smal and Large Mistuning) Madded, 2012

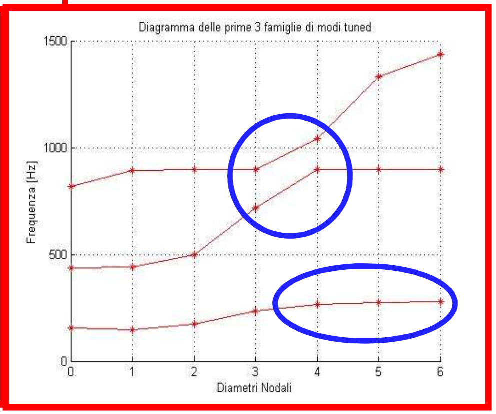
N.E.W.T. + P.R.I.M.E. reduction tecniques

Idea The mistuning pattern is added directly to the Reduced Order Model for Monte Carlo simulations

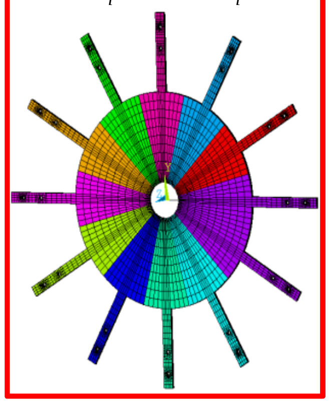
Reduction techniques: numerical validation

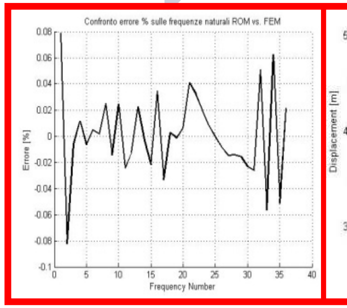
- Choice of a meaningful test case (medium-high sensitivity to mistuning)
- Full FE model generation
- Reduced Order Model (ROM) generation
- Comparison of the modal parameters and FRF in TUNED conditions
- Generation of a real-like mistuning pattern (zero mean and small st. dev. (around 3%)
- Introduction of the mistuning pattern in the full FE model and in the ROM
- Comparison of the modal parameters and FRF in MISTUNED conditions

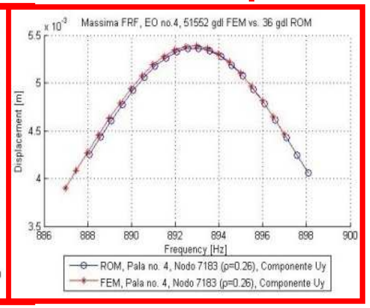




$$E_i = E^o + \Delta E_i$$





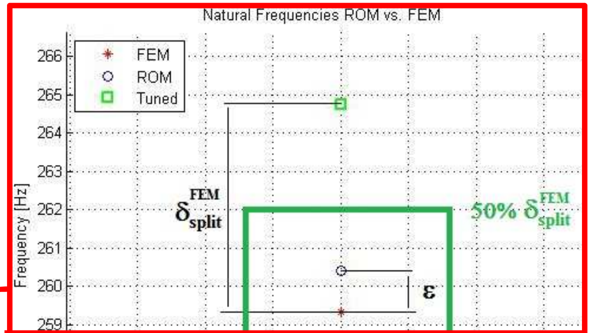


Numerical techniques to take into account mistuning: a literature review

Reduction techniques: parameter for the comparison

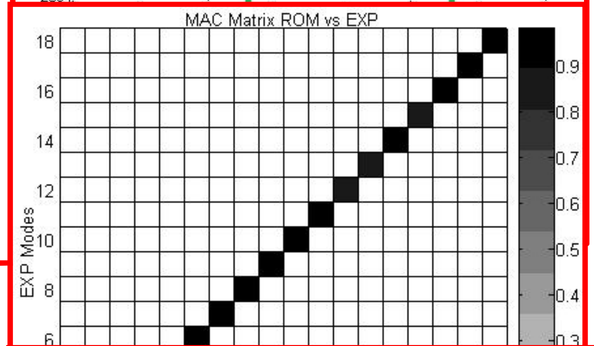
NATURAL FREQUENCIES

- Absolute percentage error: $\varepsilon = \left| \dfrac{f_{ROM} - f_{FEM}}{f_{FEM}} \right| \cdot 100$
- Frequency split error: $\varepsilon_{split}^{FEM} = \varepsilon - \left| \delta_{split}^{FEM} / 2 \right|$



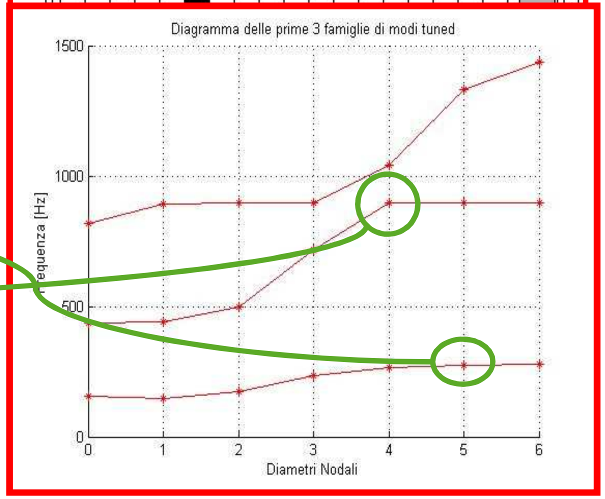
MODE SHAPES

- M.A.C. $MAC = \dfrac{\left| \phi^T \cdot \psi^* \right|}{\left(\phi^T \cdot \phi^*\right)\left(\psi^T \cdot \psi^*\right)}$



FORCED RESPONSE

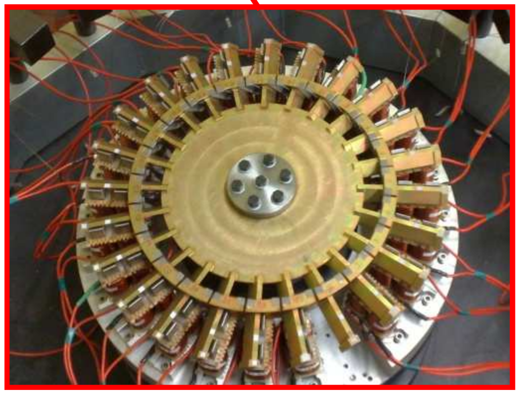
- Engine order 5 out-of-plane
- Engine order 4 in-plane
- Amplification factor: $A = x_{MAX}^{mistuned} / x_{MAX}^{tuned}$



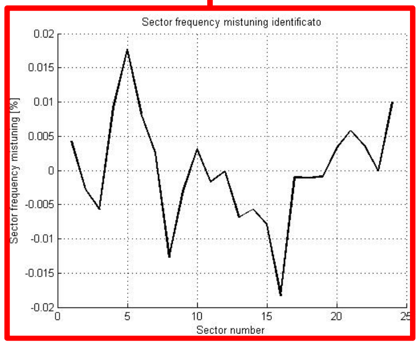
Numerical techniques to take into account mistuning: a literature review

Reduction techniques: experimental validation

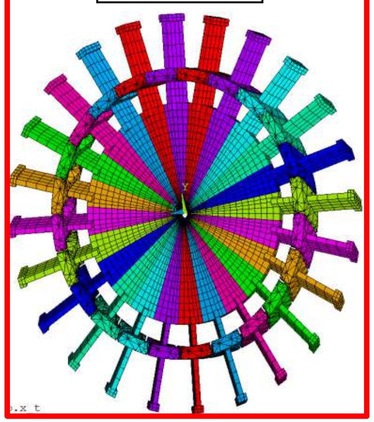
- Choice of a meaningful test case (medium-high sensitivity to mistuning)
- Hammer test, acquisition and identification of the modal parameters
- Full FE model generation
- Construction of the mistuning pattern for the full FE model
- full FE model "Updating" with the mistuning pattern, ROM generation
- Introduction of the mistuning pattern for the ROM
- Comparison of the modal parameters between ROM/FEM/Experimental



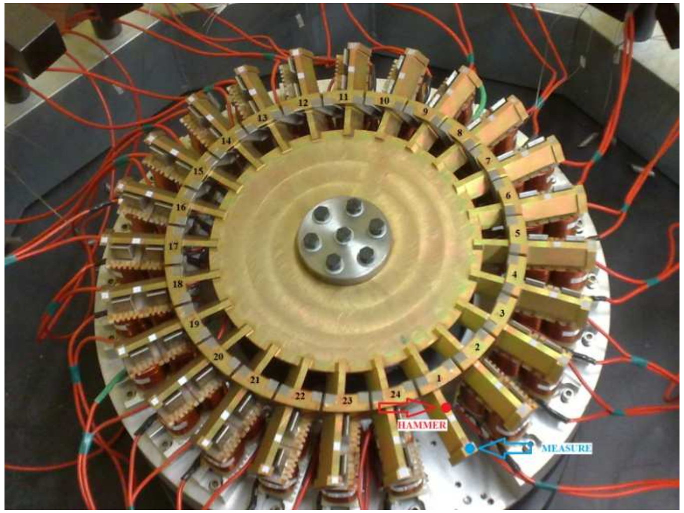
FMM ID ⇒



$E_i = E^o + \Delta E_i$

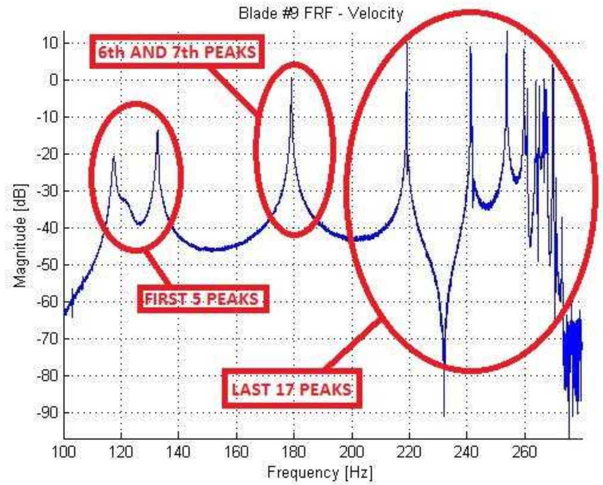


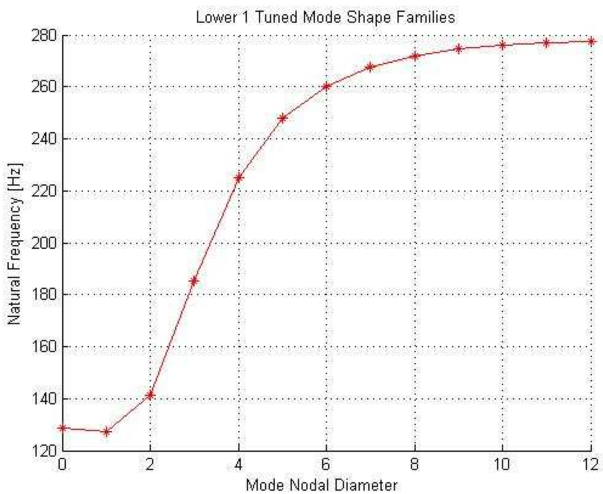
Measurement – Hammer-test



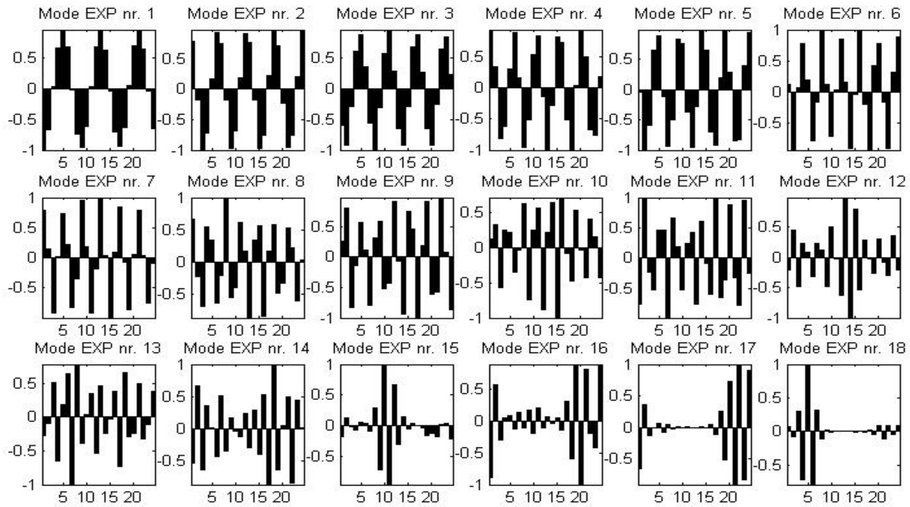
ACQUISITION
- LDV

MODAL IDENTIFICATION
- MDOF N.L.L.S. Method





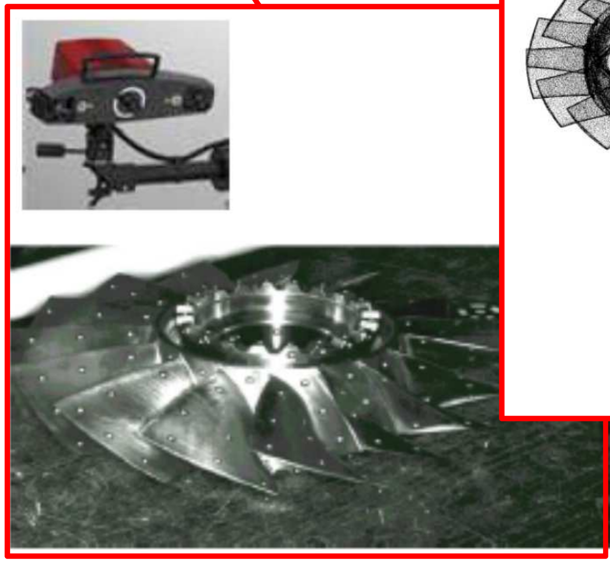
Frequencies [Hz]
179.08
219.07
219.24
241.27
242.04
253.43
253.72
259.73
260.82
263.83
264.94
266.57
266.89
267.69
269.70
269.81
270.87
272.95

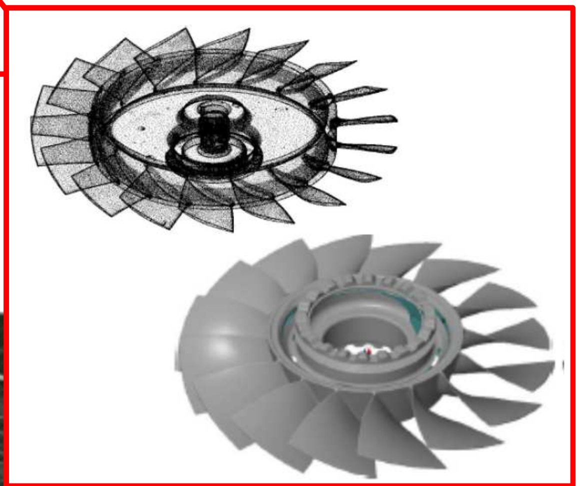


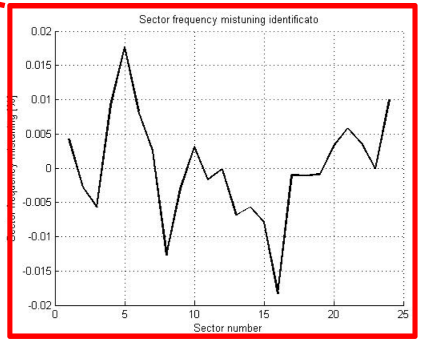
Numerical techniques to take into account mistuning: a literature review

Reduction techniques: experimental validation (alternative)

1. Choice of a meaningful test case (medium-high sensitivity to mistuning)
2. Tesselated Scan Data (TSD)
3. Full mistuned FE model generation
4. Construction of the mistuning pattern for the ROM model
5. ROM generation
6. Introduction of the mistuning pattern for the ROM
7. Comparison of the modal parameters between ROM/FEM/Experimental







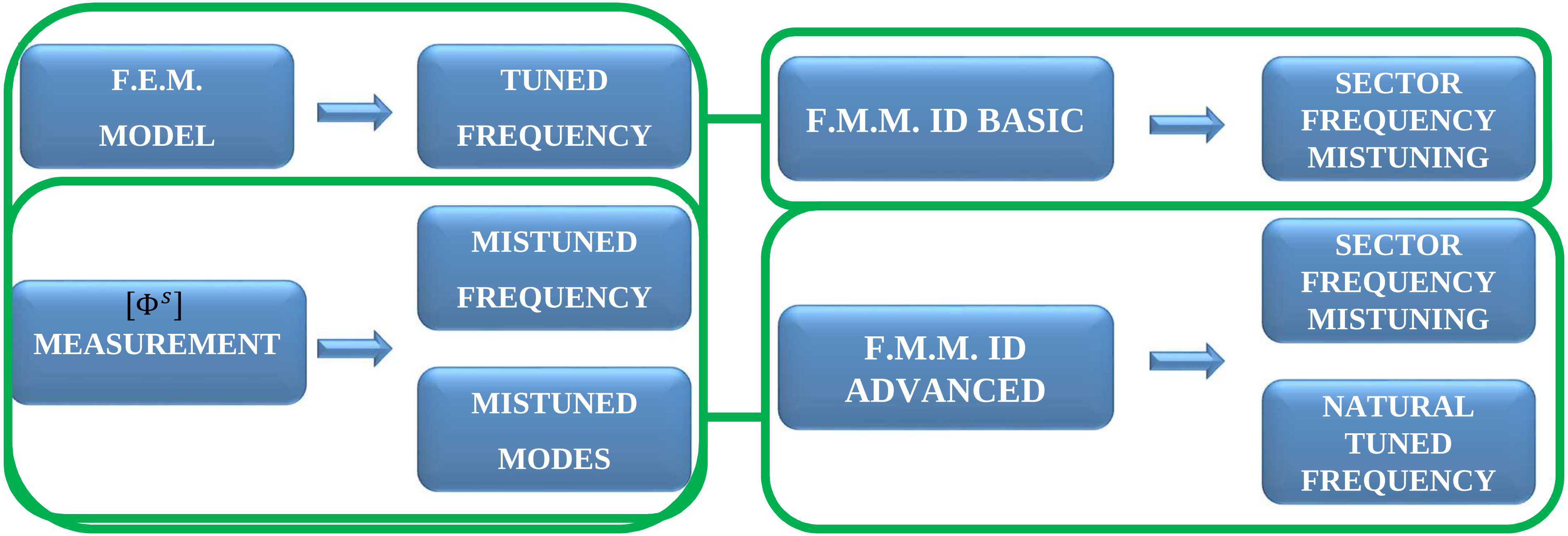
Jean de Cazenove , Scott Cogan Moustapha Mbaye (2013)

Alexander A. Kaszynski Joseph A. Beck, Jeffrey M. Brown (2014)

Numerical techniques to take into account mistuning: a literature review

F.M.M. ID is an identification technique for mistuned bladed disks developed at the Carnegie Mellon University (Griffin). It can be used for:

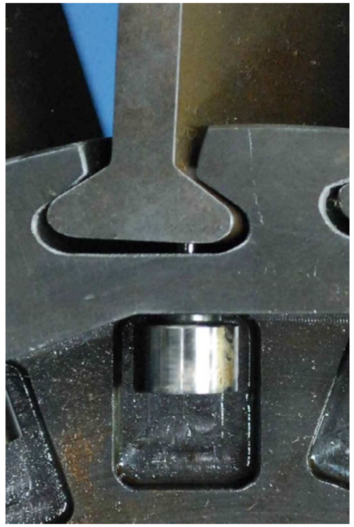
1. Isolated modal families;
2. Blade dominated modes.



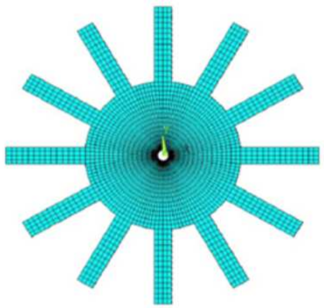
The accuracy of the mistuning pattern identified by means of the F.M.M. ID strongly depends on the accuracy of the identified modal parameters in the experiments.

Numerical techniques to take into account mistuning in the presence of friction contacts

Blade root joint as a stiffness affected by mistuning: I.M.M. ROM

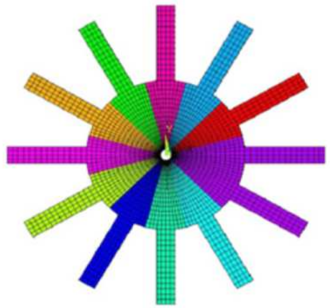


Tuned bladed disks



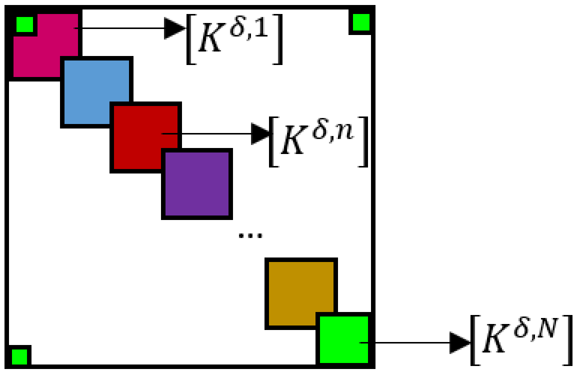
+

Perturbations of the single sectors



$[\Delta M], [\Delta K]$

$[\Delta K] =$

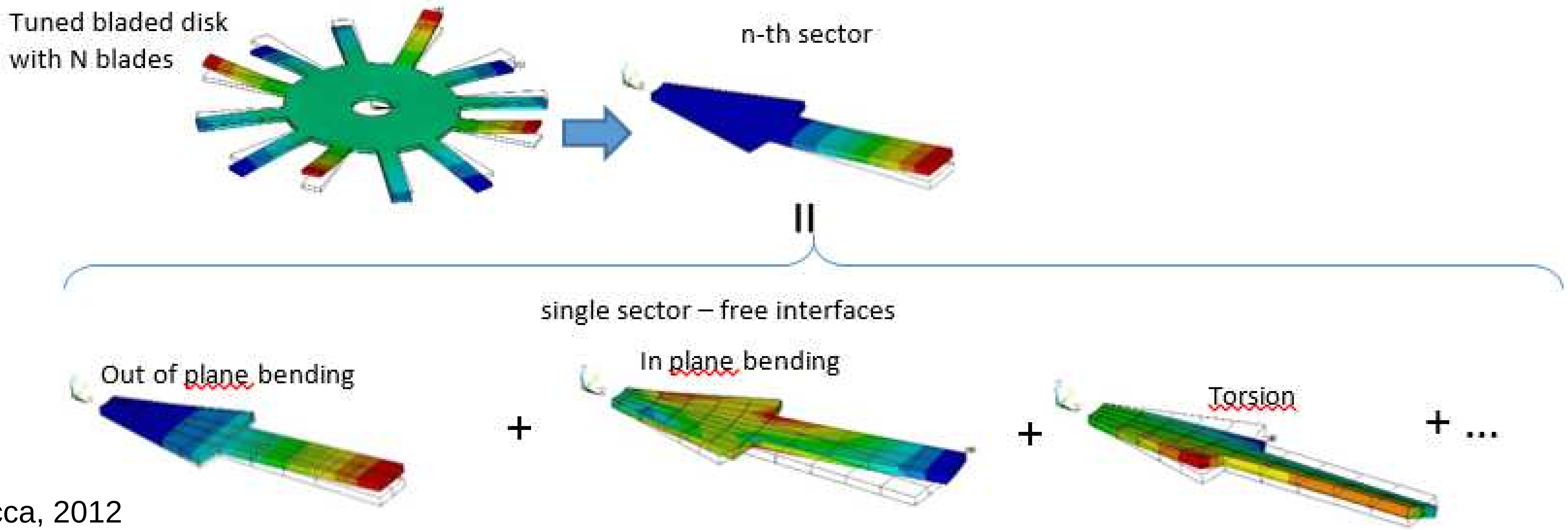


$[M], [K]$ Tuned system

$[\Lambda^s]$ Tuned eigenvalues

$[\Phi^s]$ Tuned eigenvectors

Key idea:

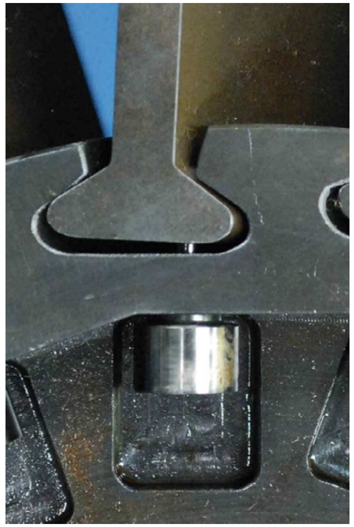


$[\Phi^{0,n}]$

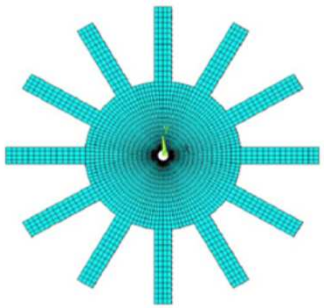
Vargiu, Firrone, Zucca, 2012

Numerical techniques to take into account mistuning in the presence of friction contacts

Blade root joint as a stiffness affected by mistuning



Tuned bladed disks

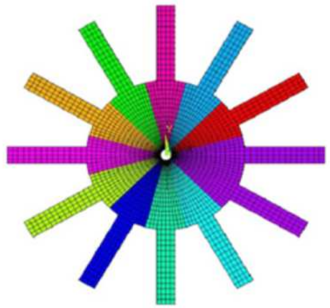


$[M], [K]$ Tuned system

$[\Lambda^s]$ Tuned eigenvalues

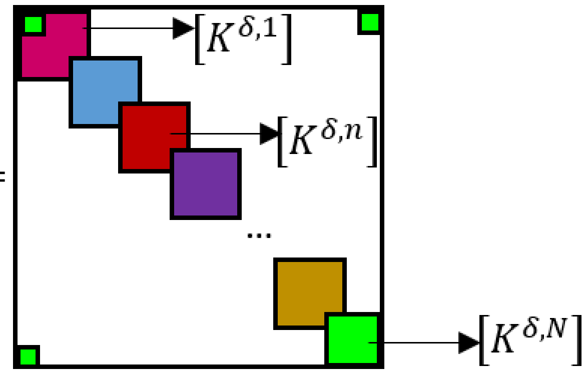
$[\Phi^s]$ Tuned eigenvectors

Perturbations of the single sectors



$[\Delta M], [\Delta K]$

$[\Delta K] =$



Prohibitive time cost
for computation

$$[M^{mist}] = [M] + [\Delta M] \;\Rightarrow\; [\Phi^s]^T[M^{mist}][\Phi^s] = \mu^{rom} = I + \boxed{[\Phi^s]^T[\Delta M][\Phi^s]}$$

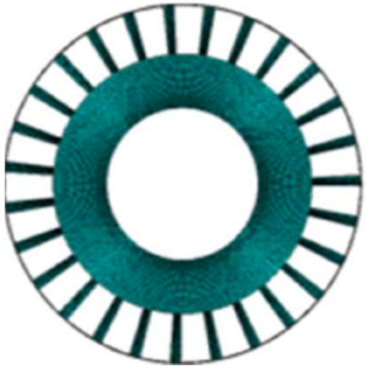
$$[K^{mist}] = [K] + [\Delta K] \;\Rightarrow\; [\Phi^s]^T[K^{mist}][\Phi^s] = k^{rom} = \Lambda^s + \boxed{[\Phi^s]^T[\Delta K][\Phi^s]}$$

$$[\Phi^s]^T[\Delta K][\Phi^s] = \sum_{n=1,..,N} [\Phi^{s,n}]^T[K^{\delta,n}][\Phi^{s,n}] \approx \sum_{n=1,..,N} [\Phi^{0,n}q^n]^T\,[K^{\delta,n}][\Phi^{0,n}q^n] =$$

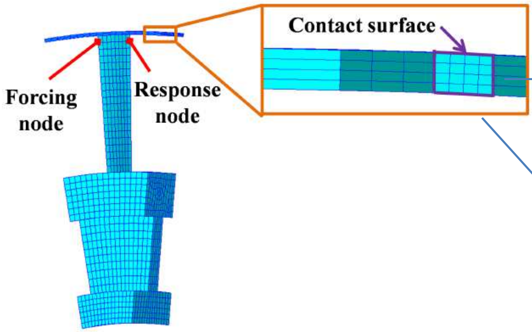
$$= \sum_{n=1,..,N} [q^n]^T[\Phi^{0,n}]^T\,[K^{\delta,n}][\Phi^{0,n}]\,[q^n] = \sum_{n=1,..,N} [q^n]^T\; diag_{p=1,\dots,P}(\delta_p^n \cdot \lambda_p)[q^n]$$

Vargiu, Firrone, Zucca, 2012

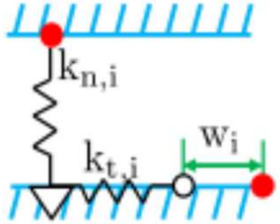
Shroud contact: stiffness affected by mistuning and nonlinear forces



a CMS – CB ROM is produced

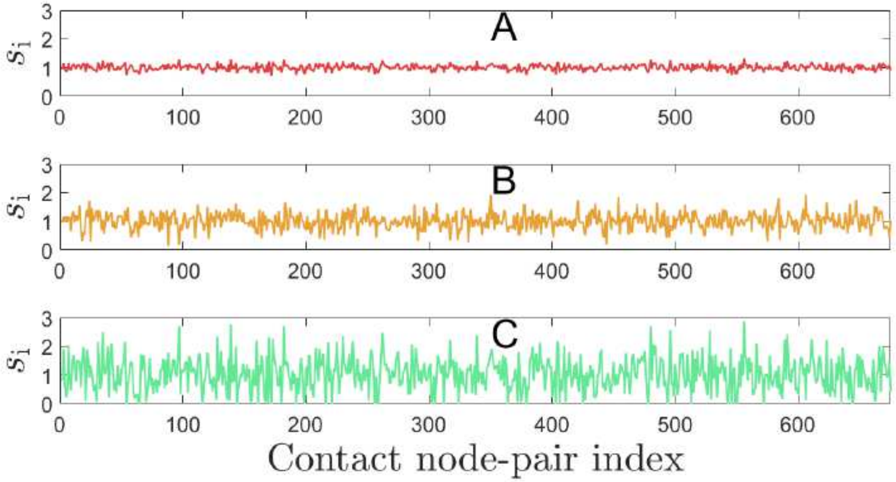


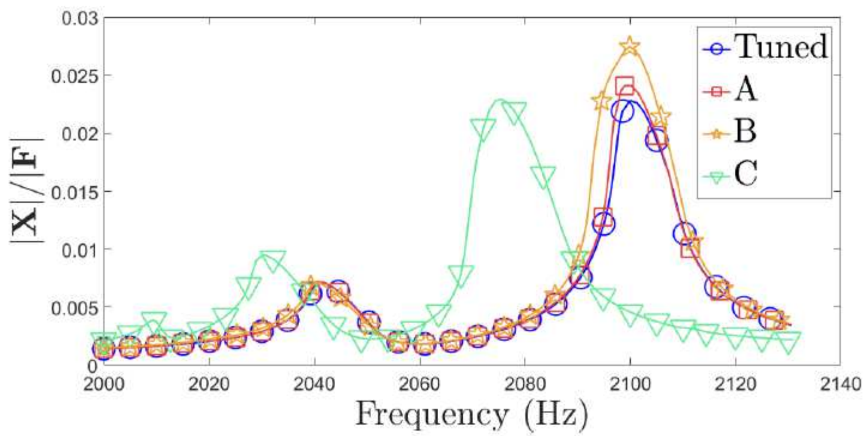
Master nodes



Friction coefficient

Mistuning pattern on contact stiffness





RESPONSE TO EO 12 EXCITATION

Mitra, Zucca, Epureanu, 2016

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EXPERTISE

POLITECNICO DI TORINO
Department of Mechanical and Aerospace Engineering

KICK-OFF TRAINING

TUM - Garching-bei-München
2018 Jan 8-12

Forced response measurement of bladed disks

Teresa Berruti, Christian Firrone
Politecnico di Torino
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Outline

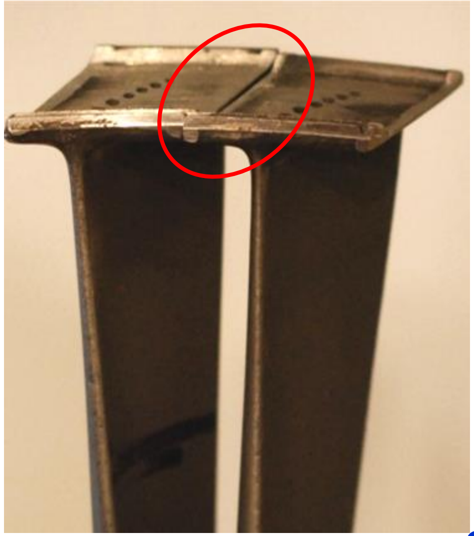
- Typical friction contacts in bladed disks
- The experimental validation by bladed disk test rigs
- Example of test rigs with bladed disks in the world
- Some important diagrams
- The bladed disk test rigs at Politecnico di Torino

Typical friction contacts in bladed disks

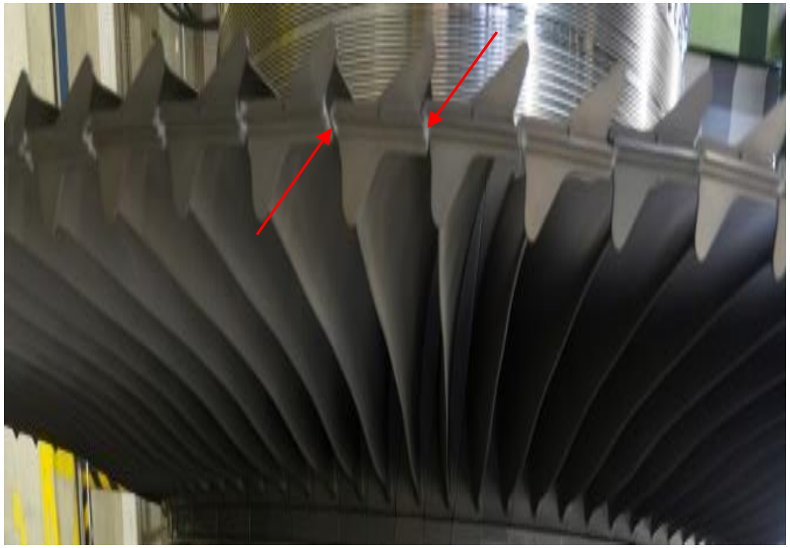
Typical friction contacts in bladed disks

Blade to blade

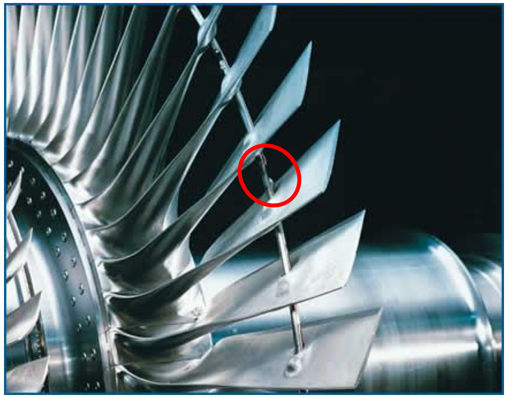
Shroud

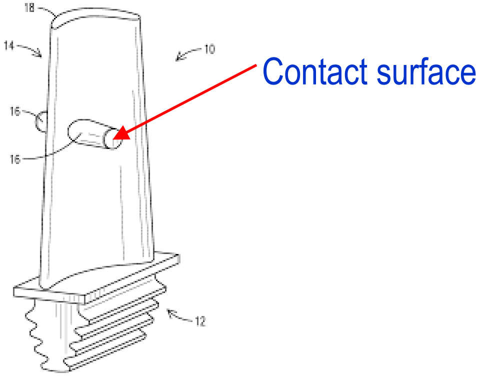


Shroud with interlocking



Snubbers





Typical friction contacts in bladed disks

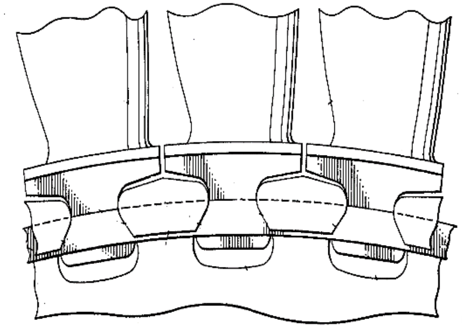
Blade to disk

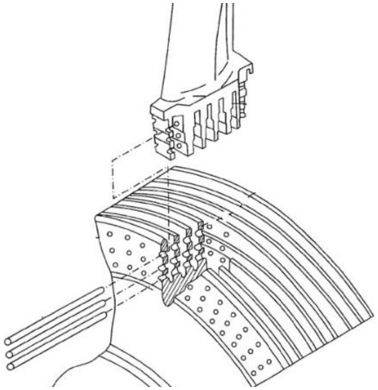
fir tree

dove tail

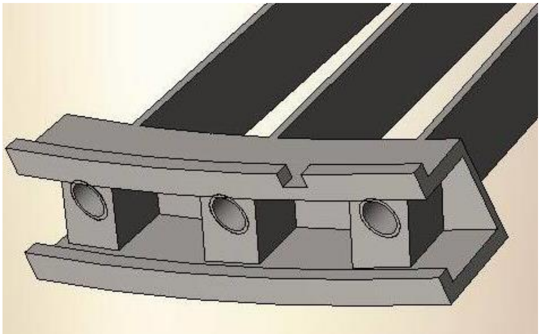
pinned root

Blade root





Hooks



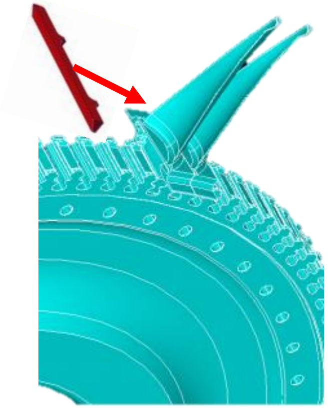
Typical friction contacts in bladed disks

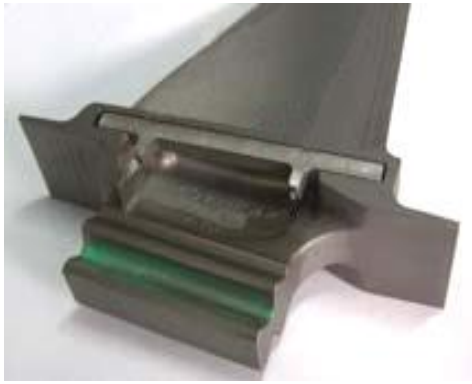
Blade to damper

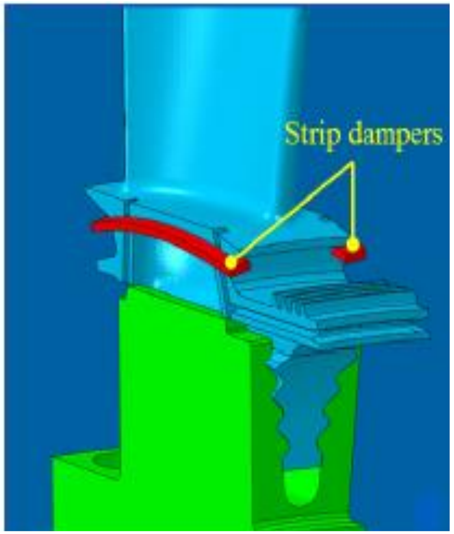
Rigid damper

Flexible damper /strip damper

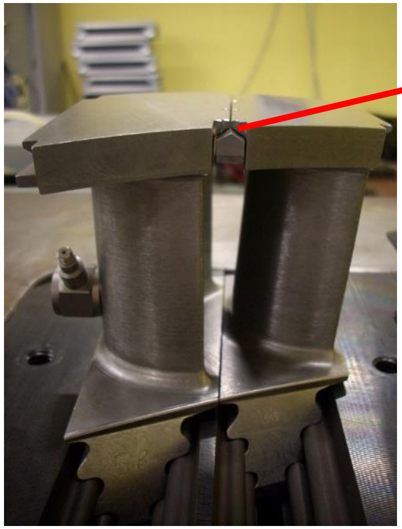
Underplatform dampers







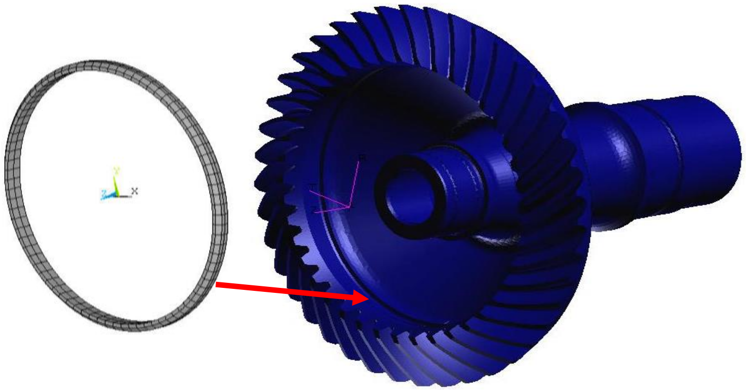
Tip dampers



Typical friction contacts in bladed disks

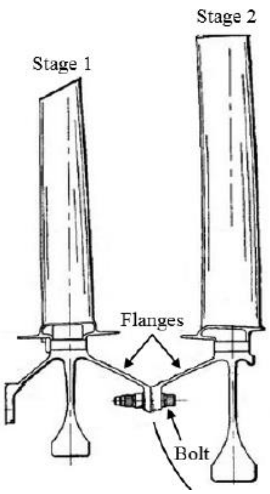
Disk to damper

Ring damper



Disk to Disk

Bolted flanges between disks



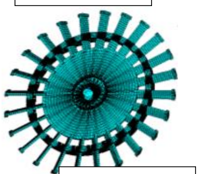
The experimental validation by bladed disk test rigs

Prediction of non linear dynamic of bladed disks with friction damping

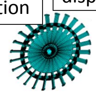
Example : case of disk with underplatform damper

Numerical calculation

Disk FE model



condensation



displacements

nonlinear forces

•Damper model
•Kynematic model
•Contact model

time domain

$$\mathbf{M}\ddot{\mathbf{u}}^*(t) + \mathbf{C}\dot{\mathbf{u}}^*(t) + \mathbf{K}\mathbf{u}^* = \mathbf{F}_E(t) - \mathbf{F}_C(t)$$

$$\mathbf{M}_D\ddot{\mathbf{h}}(t) = \mathbf{F}_C(t) + \mathbf{F}_{E-D}$$

Damper-rigid body or elastic model

frequency domain

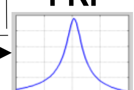
$$\mathbf{D}(\omega)^{(n)}\mathbf{u}^{*(n)} = \mathbf{F}_E^{(n)} - \mathbf{F}_C^{(n)}$$

$$\mathbf{D}_D(\omega)^{(n)}\mathbf{h}^{(n)} = \mathbf{F}_{E-D}^{(n)} + \mathbf{F}_C^{(n)}$$

Solution by Multi Harmonic Balance Method

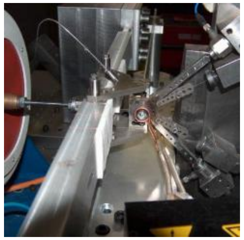
Hysteresis loop

FRF



Experiments

Contact parameters



1st Experimental validation

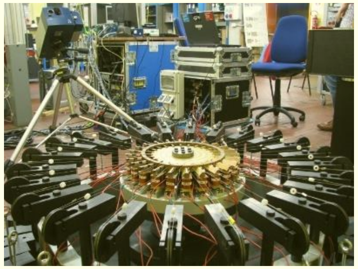
1

Experimental validation with real disk

3

Experimental validation with dummy disk in lab condition

2

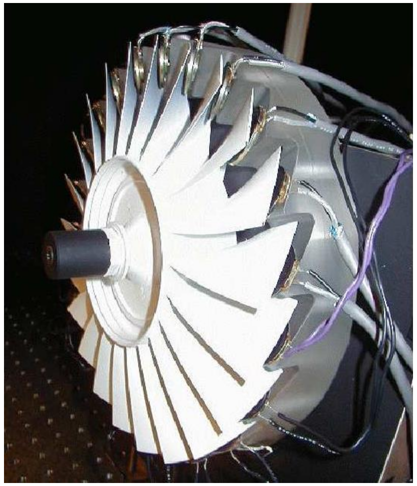


Example of test rigs with bladed disks in the world

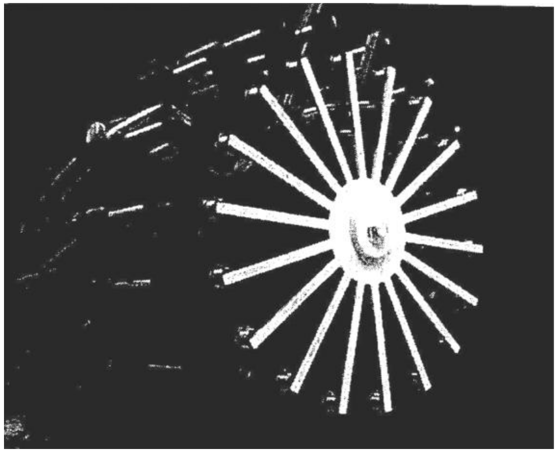
Example of test rigs for bladed disks in the world

Static rigs (not rotating)

Pierre C., Ceccio S.L., et al.
University of Michigan, 2003.

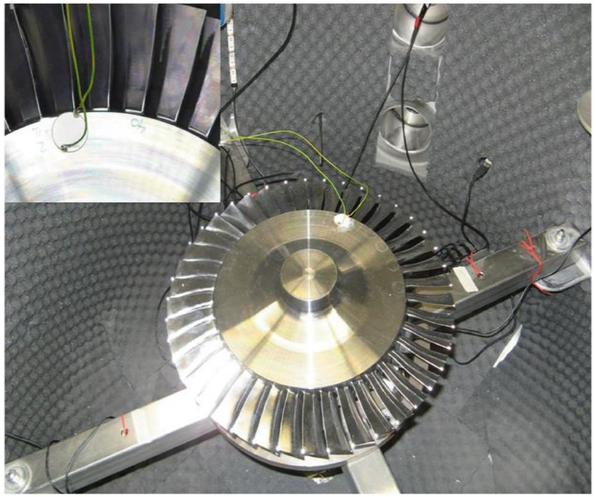


Jones K.W. , Cross C.J.
US Air Force Res.Lab. 2003



Strehlau U., Kuhhorn A.
Brandenburg Univ. of Tech. , 2010

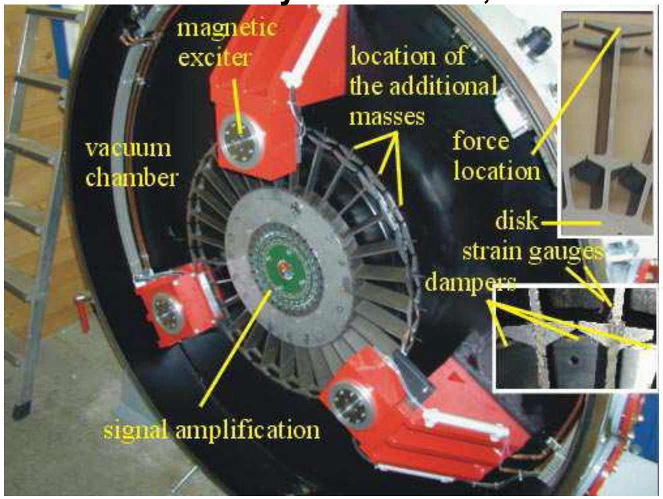
Beirow B. et al.
Brandenburg Univ. of Tech., 2018



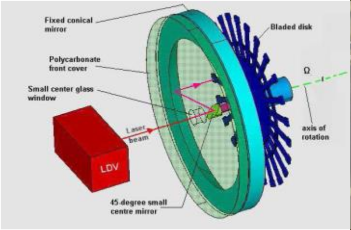
Example of test rigs for bladed disks in the world

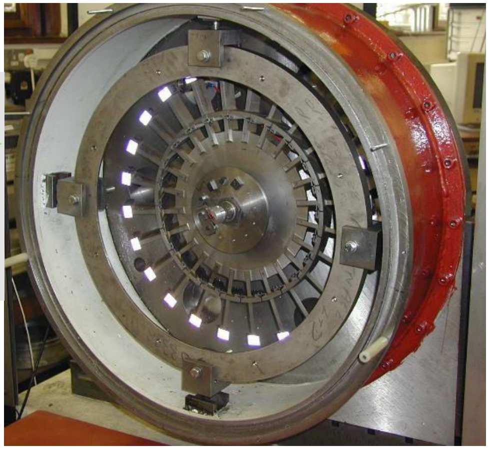
Rotating rigs

Gotting F., Sextro W., Panning L., Popp K., University of Hannover, 2004.

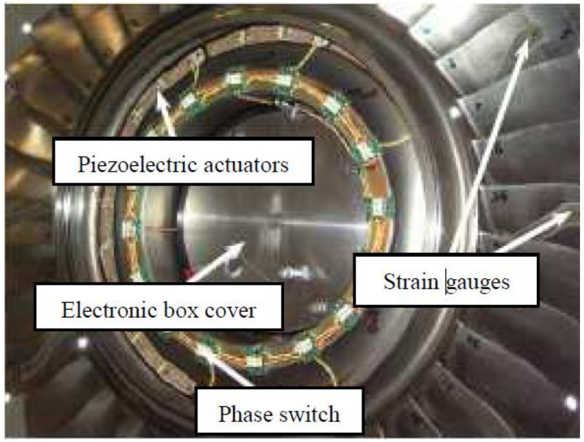


Ewins J., et al. Imperial College London, 2004

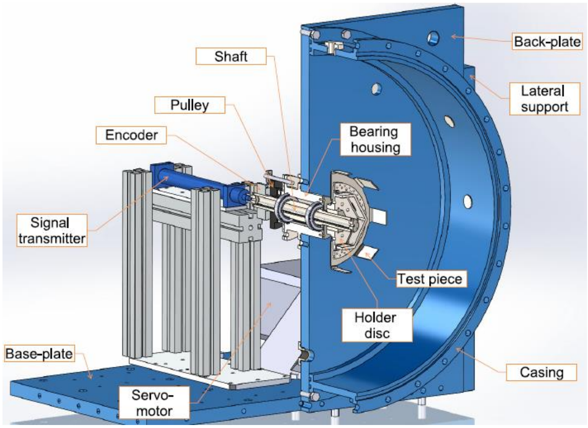


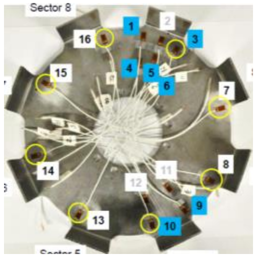


Gibert C., Thouverez F. et al. Lab. of Tribology and Sys. Dynamics, Ecole Centrale de Lyon, 2010



Ruffini, V., Schwingshackl, C. W., and Green, J. S., Imperial College London, 2017





Main common features of the rigs

Static rigs (not rotating)

Rotating rigs

The disk is simplified in order to highlight a particular phenomenon

Static rigs (not rotating)

- Travelling excitation force

- Non contact excitation systems (acoustic speakers, electromagnet) or with low impact on the structure (piezoelectric actuators)

- Measurement of the response: laser vibrometer or strain gages

Manly used to validate mistuning model (linear cases)

Rotating rigs

- Excitation of the blades by permanent magnets (synchronous excitation) , electromagnet or piezo actuators (also asynchronous excitation)

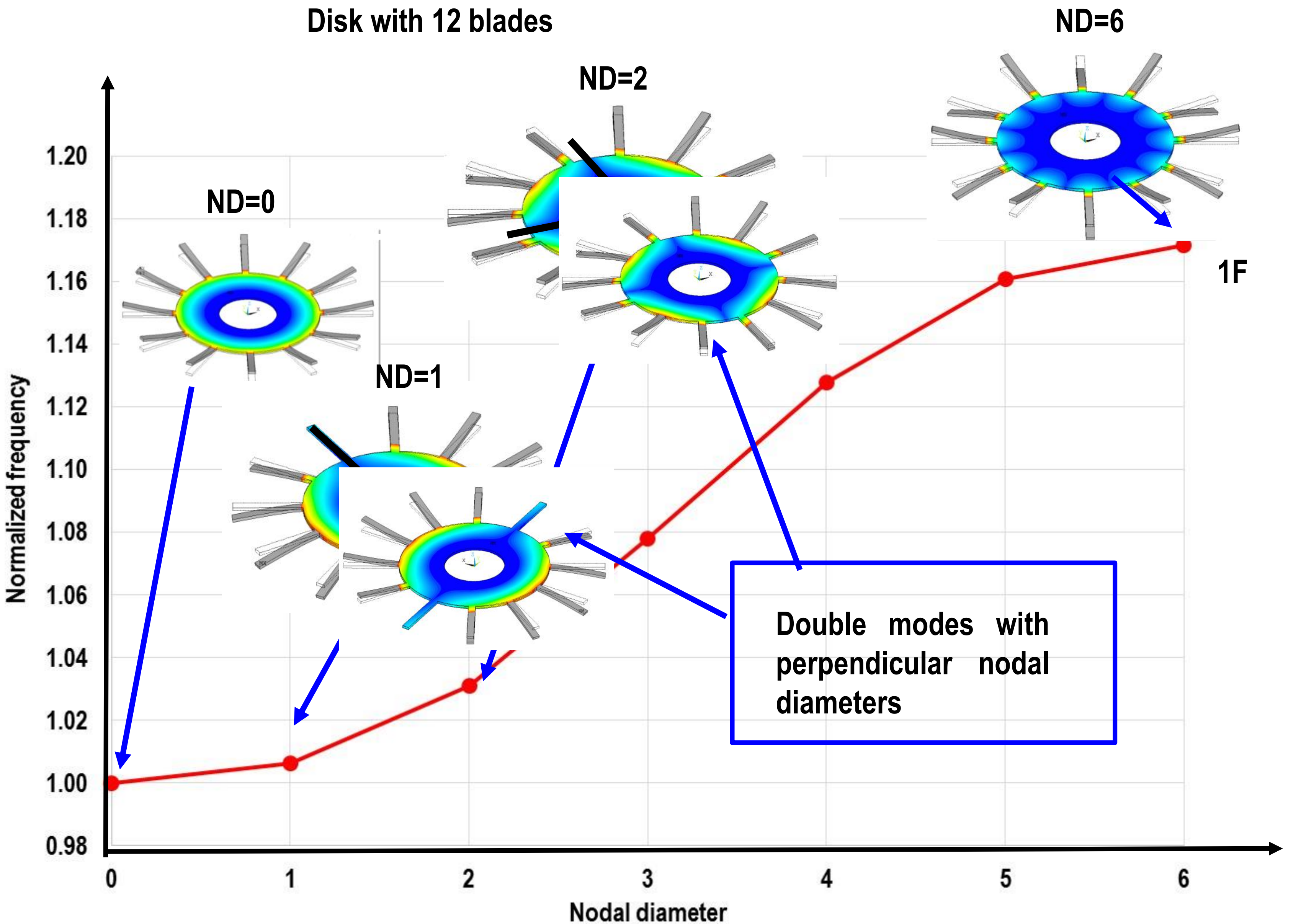
- Most of them rotates in vacuum

- Measurement of the response: by strain gages or tip timing system

Manly used to validate mistuning models (linear cases), interaction with mistuning and rotation speed (Coriolis effect) , damping characterization (friction damping devices)

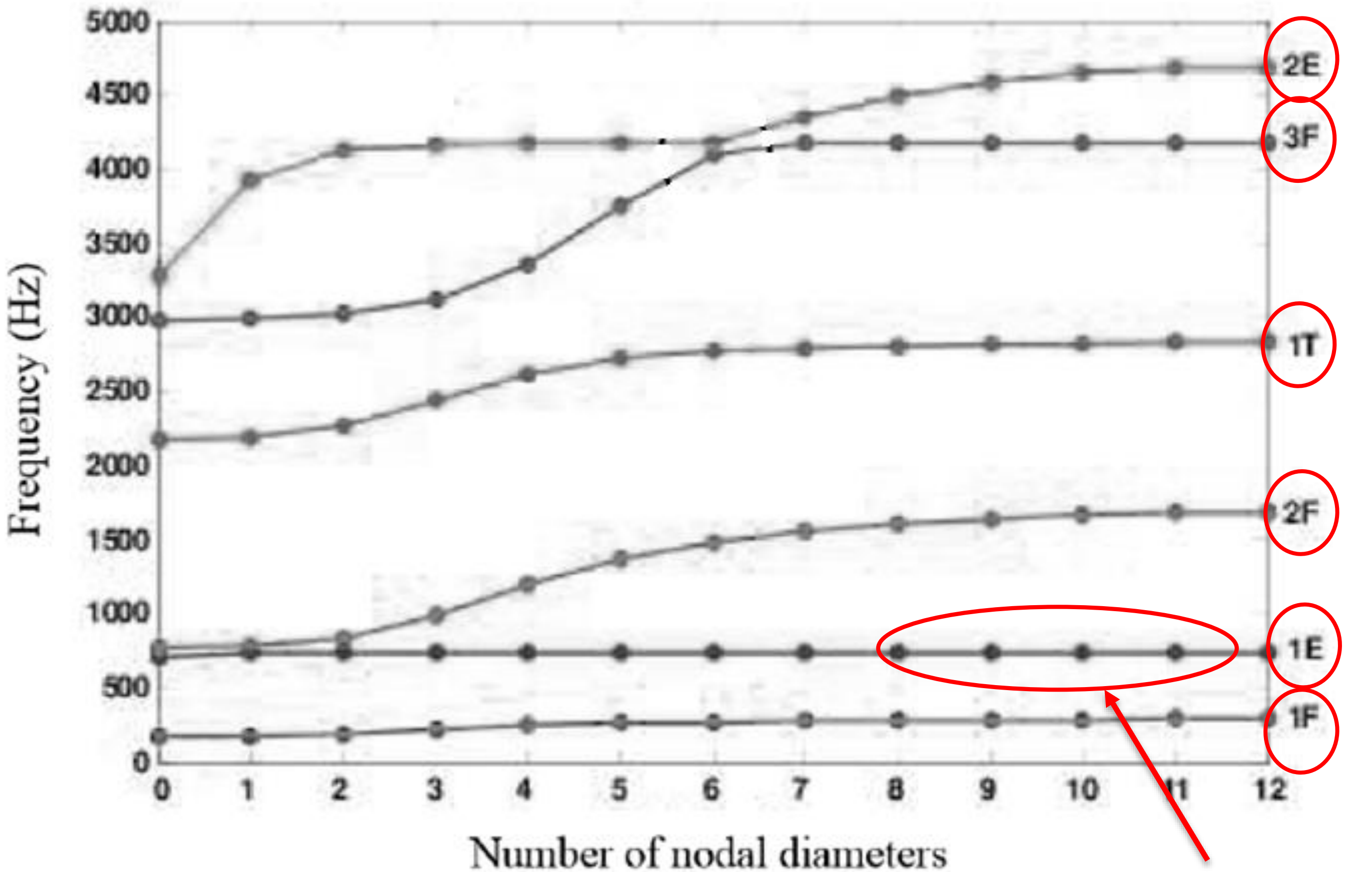
Some important diagrams

The Frequency-Nodal Diameter diagram (FREND)



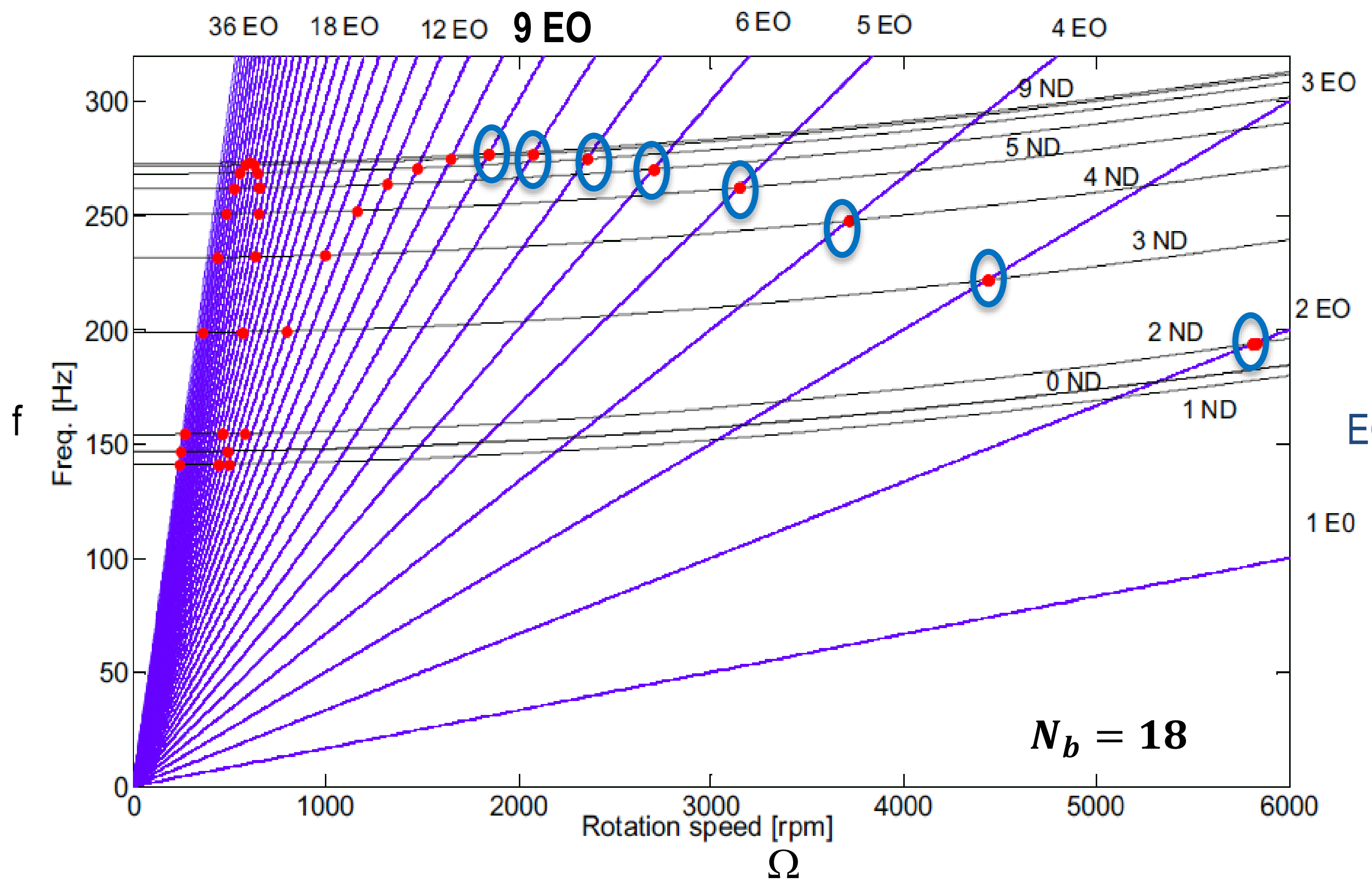
The Frequency-Nodal Diameter diagram (FREND)

Disk with 24 blades different modal families



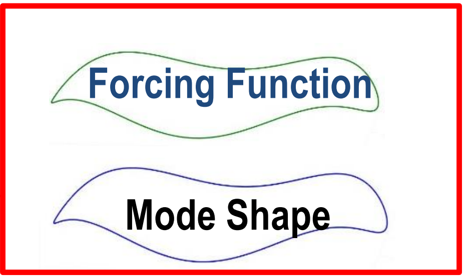
High modal density region

The Campbell diagram



EO = ND

- Resonance :
  - frequency of the force = natural frequency $EO \cdot \Omega = f$
  - shape of the force = shape of the excited mode



The Campbell diagram

Resonance when

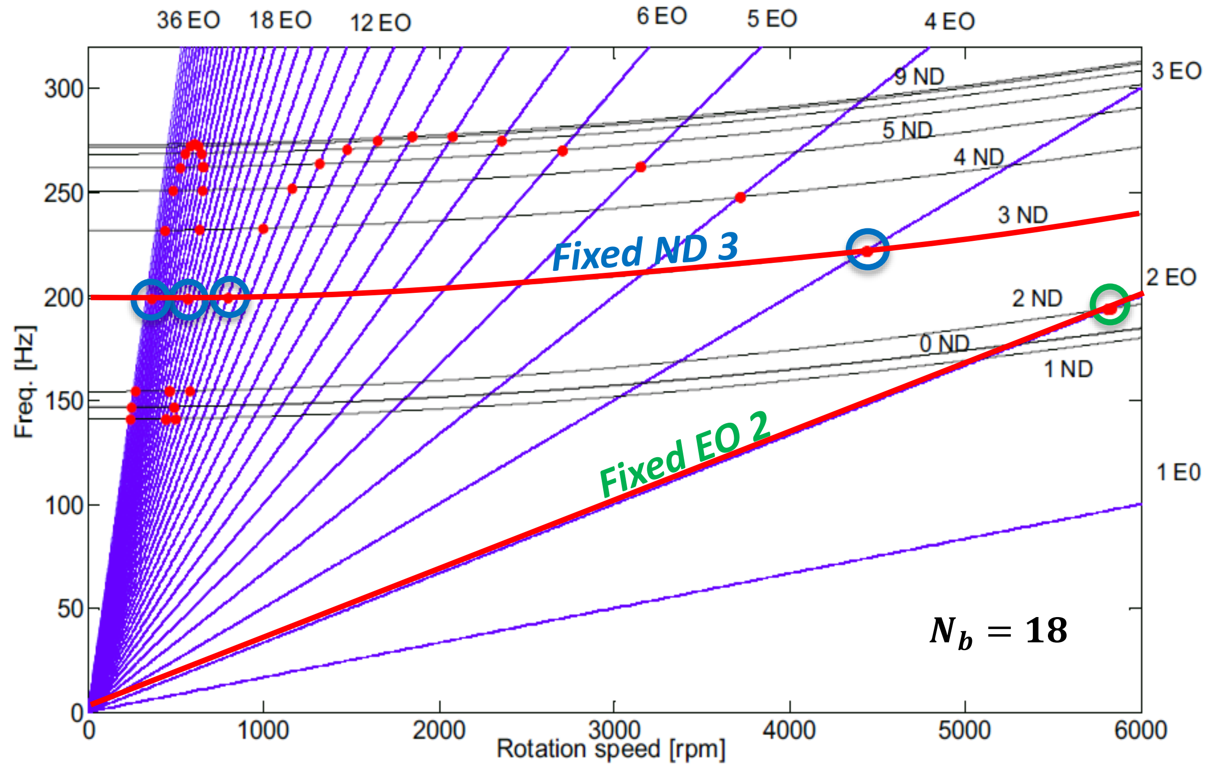
- $\mathbf{EO} = \mathbf{ND}$

but also

- $\mathbf{EO} = \mathbf{j} \cdot \mathbf{N_b} \pm \mathbf{ND}$

By looking at a generic Campbell diagram it can be noticed that each mode corresponding to a certain ND line intersects different EO lines

..while each EO line intersects just one ND line.



Example

Fixed EO

$$2 = 0 \cdot 18 + 2$$ ✓

$$2 = 1 \cdot 18 - 16$$ ✗

$$2 = 2 \cdot 18 - 34$$ ✗

⋮ ✗ ✗

Only ND 2 satisfies the equation

ND 16 & ND 34 do not exist!

Fixed ND

$$3 = 0 \cdot 18 + 3$$ ✓

$$15 = 1 \cdot 18 - 3$$ ✓

$$21 = 1 \cdot 18 + 3$$ ✓

⋮ ✓ ✓

The same ND can be excited by different EOs

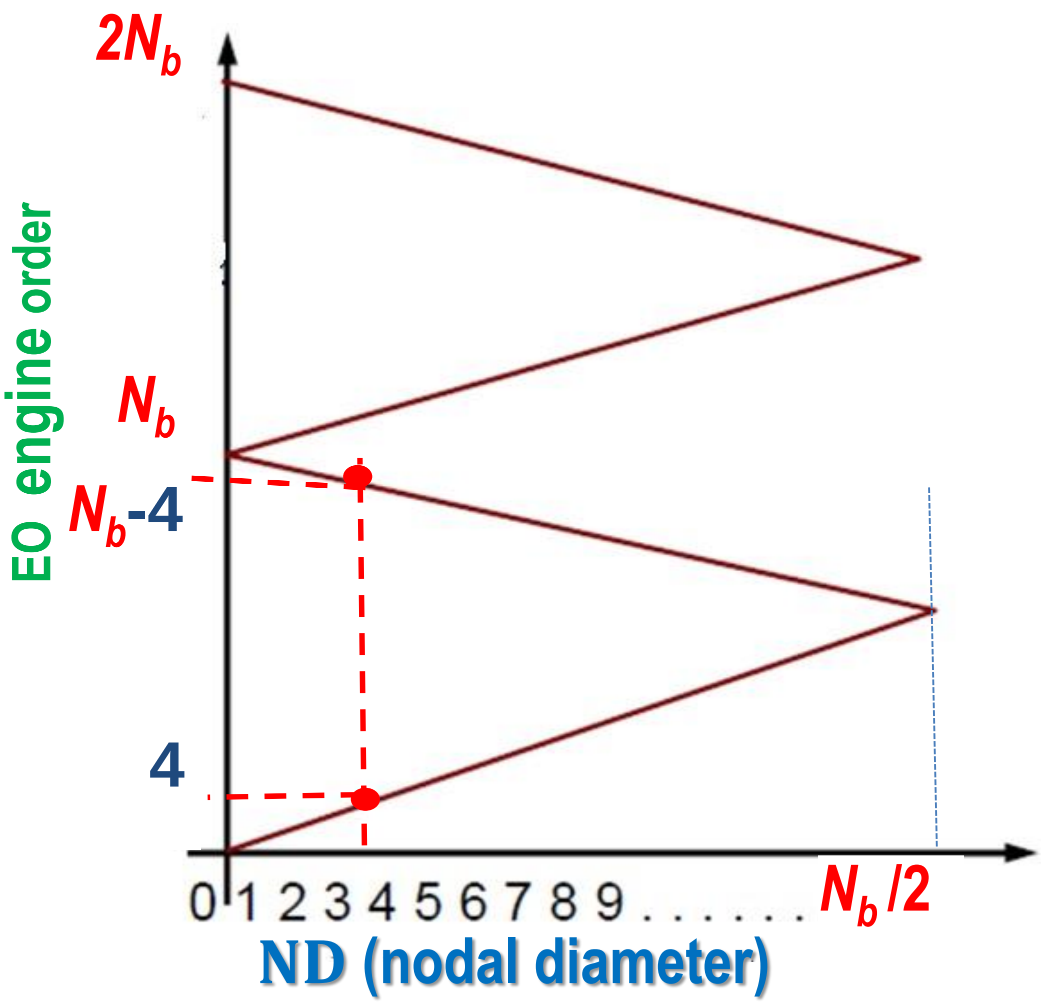
$\boldsymbol{EO}$: *Engine Order*

$\boldsymbol{N_b}$: *# Blades*

$\boldsymbol{ND}$: *# Nodal Diameters*

$\boldsymbol{j}$ = *0,1,2,3,...*

The Zig-Zag diagram – SAFE diagram



$$\mathbf{EO} = \mathbf{j} \cdot \mathbf{N_b} \pm \mathbf{ND}$$

Example

N_b=24 number of blades

To excite ND=4 → EO=4

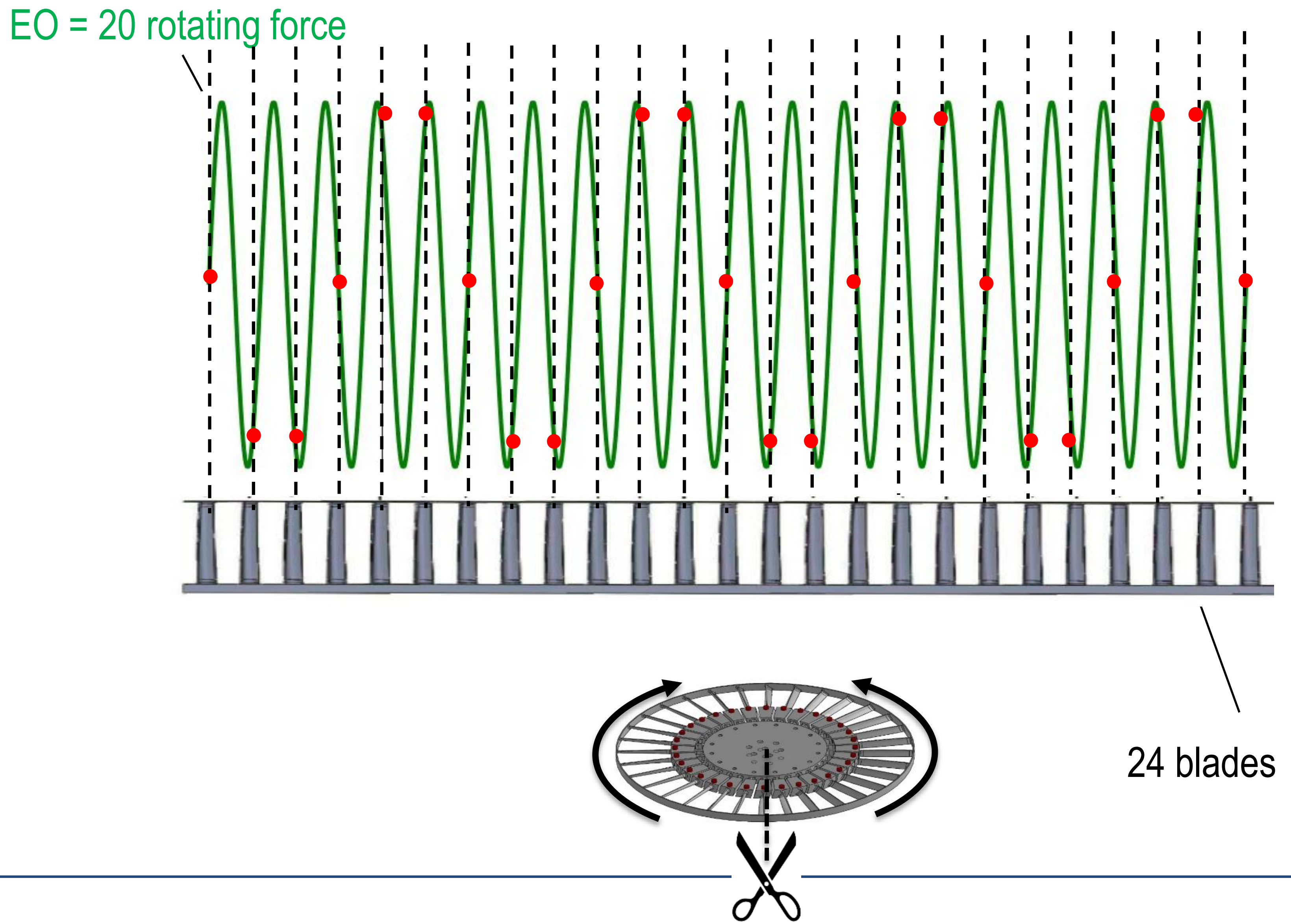
→ EO=24-4=20

Why EO=20 can excite ND=4

M.P.Singh, J. (2002). SAFE diagram- A design and reliability tool for turbine blading. Proceeding of the 17th Turbomachinery Symposium.

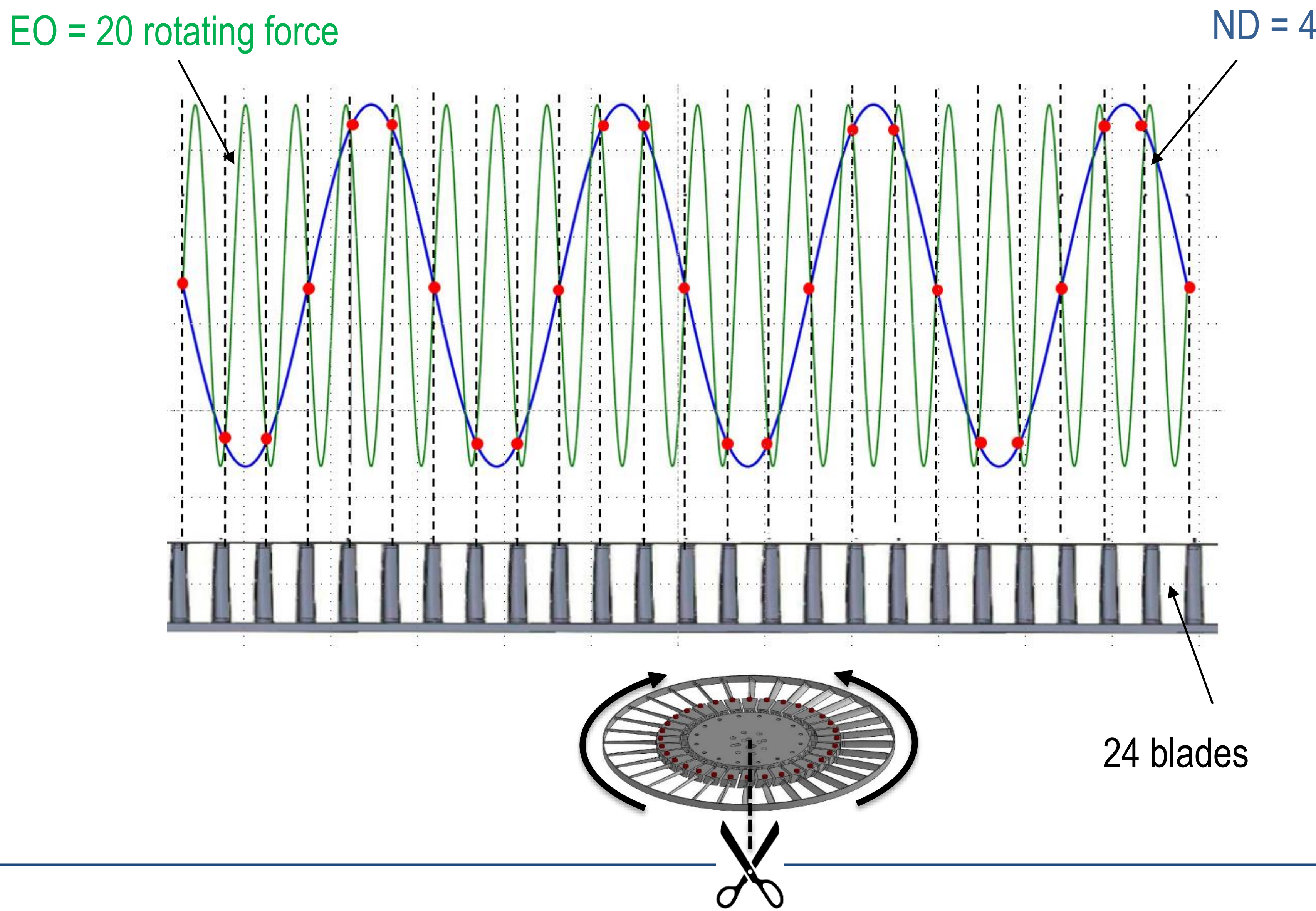
Explanation of the Zig-Zag diagram

It is do to the Aliasing phenomenon…..



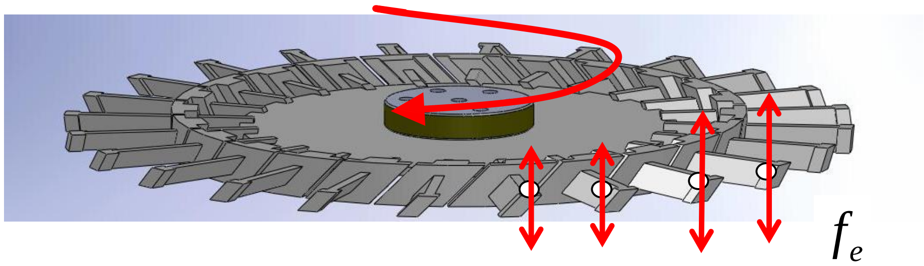
Explanation of the Zig-Zag diagram

It is do to the Aliasing phenomenon…..



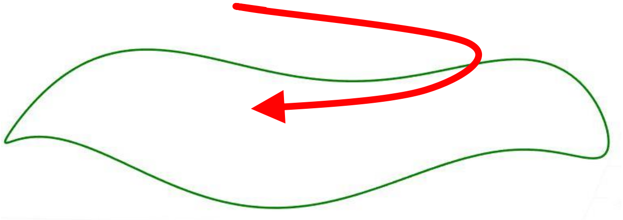
TRAVELLING WAVE EXCITATION on a NON ROTATING DISK

TRAVELLING FORCE



Travelling Excitation force

$$f_e = F_E \cos(\omega_m t + (n-1) \cdot \varphi_m)$$



$n = 1, \ldots, N_b = 24$ Number of the blade

EO engine order index

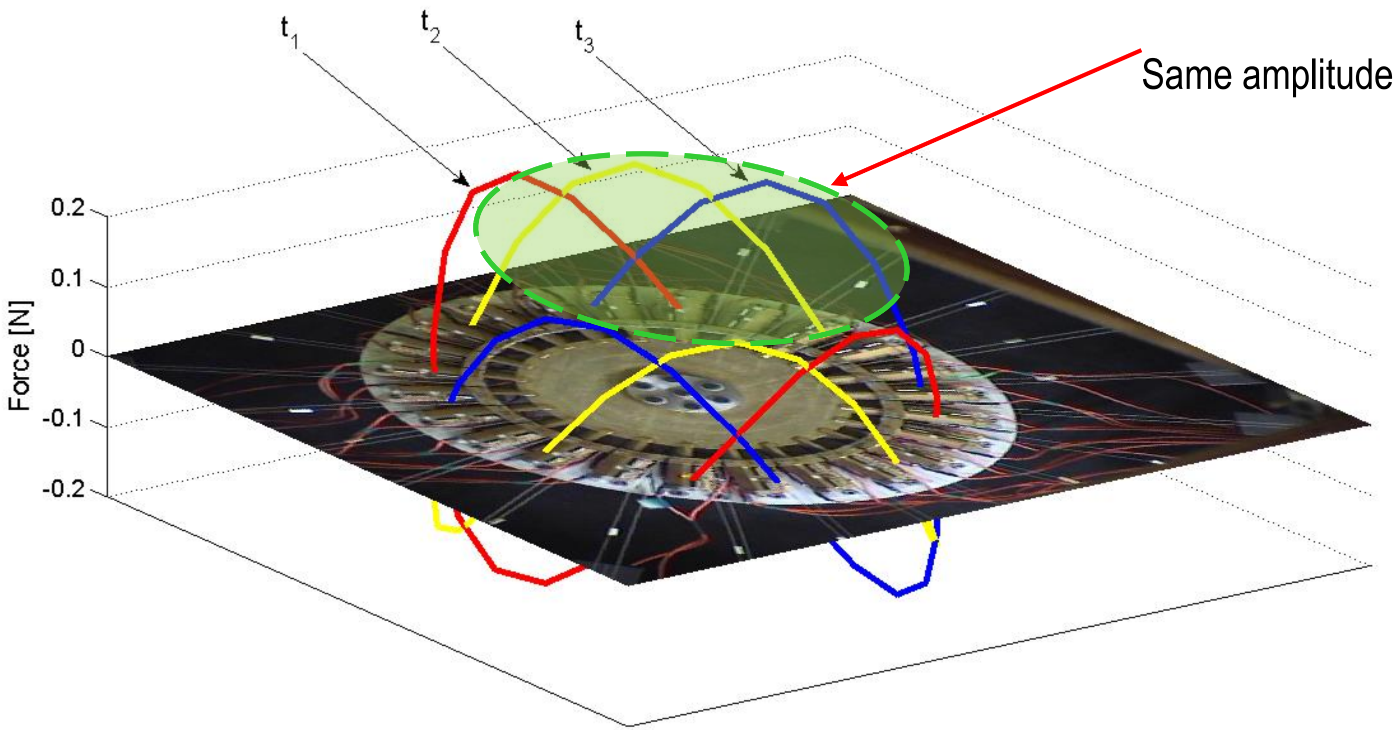
$$\varphi_m = \frac{2\pi}{N_b} EO$$ Inter Blade excitation angle

Example $EO = 0 \quad \varphi_m = 0$ $\qquad EO = \frac{N_b}{2} \quad \varphi_m = \pi$

TRAVELLING WAVE EXCITATION on a NON ROTATING DISK

$$f_e = F_e \cos(\omega_m t + (n-1)\cdot\varphi_m)$$

Plot of the force at time t_1, t_2, t_3



The force can also be written as the sum of two orthogonal waves

$$\begin{aligned} f_e &= F_e \cos(\omega_m t + (n-1)\cdot\varphi_m) \\ &= \underline{F_e \cos((n-1)\cdot\varphi_m)\cdot\cos(\omega_m t)} - \underline{F_e \mathrm{sen}((n-1)\cdot\varphi_m)\cdot\mathrm{sen}(\omega_m t)} \end{aligned}$$

TRAVELLING WAVE EXCITATION on a NON ROTATING DISK

RESPONSE TO A TRAVELLING FORCE

$$x = X\cos(\omega_m t + (n-1)\cdot\varphi_m) \qquad n = 1,\dots,N_b = 24$$

$$x = \underline{X\cos((n-1)\cdot\varphi_m)\cdot\cos(\omega_m t)} - \underline{X\mathrm{sen}((n-1)\cdot\varphi_m)\cdot\mathrm{sen}(\omega_m t)} \qquad \varphi_m = \frac{2\pi}{N_b}EO$$

EO engine order index

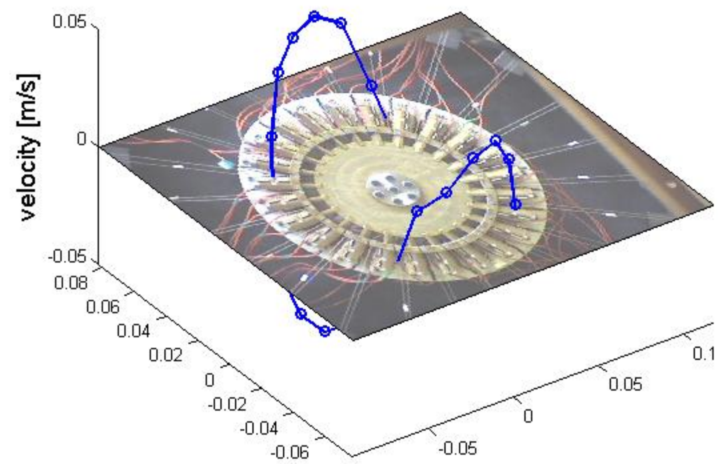
TRAVELLING WAVE EXCITATION on a NON ROTATING DISK

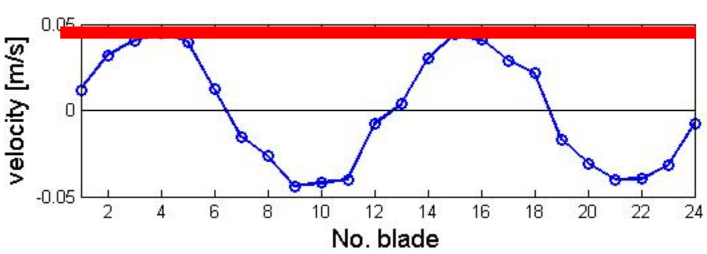
RESPONSE TO A TRAVELLING FORCE

$$x = X\cos(\omega_m t + (n-1)\cdot\varphi_m) \qquad n = 1,\dots,N_b = 24$$

$$x = X\cos((n-1)\cdot\varphi_m)\cdot\cos(\omega_m t) - X\mathrm{sen}((n-1)\cdot\varphi_m)\cdot\mathrm{sen}(\omega_m t) \qquad \varphi_m = \frac{2\pi}{N_b}EO$$

EO engine order index

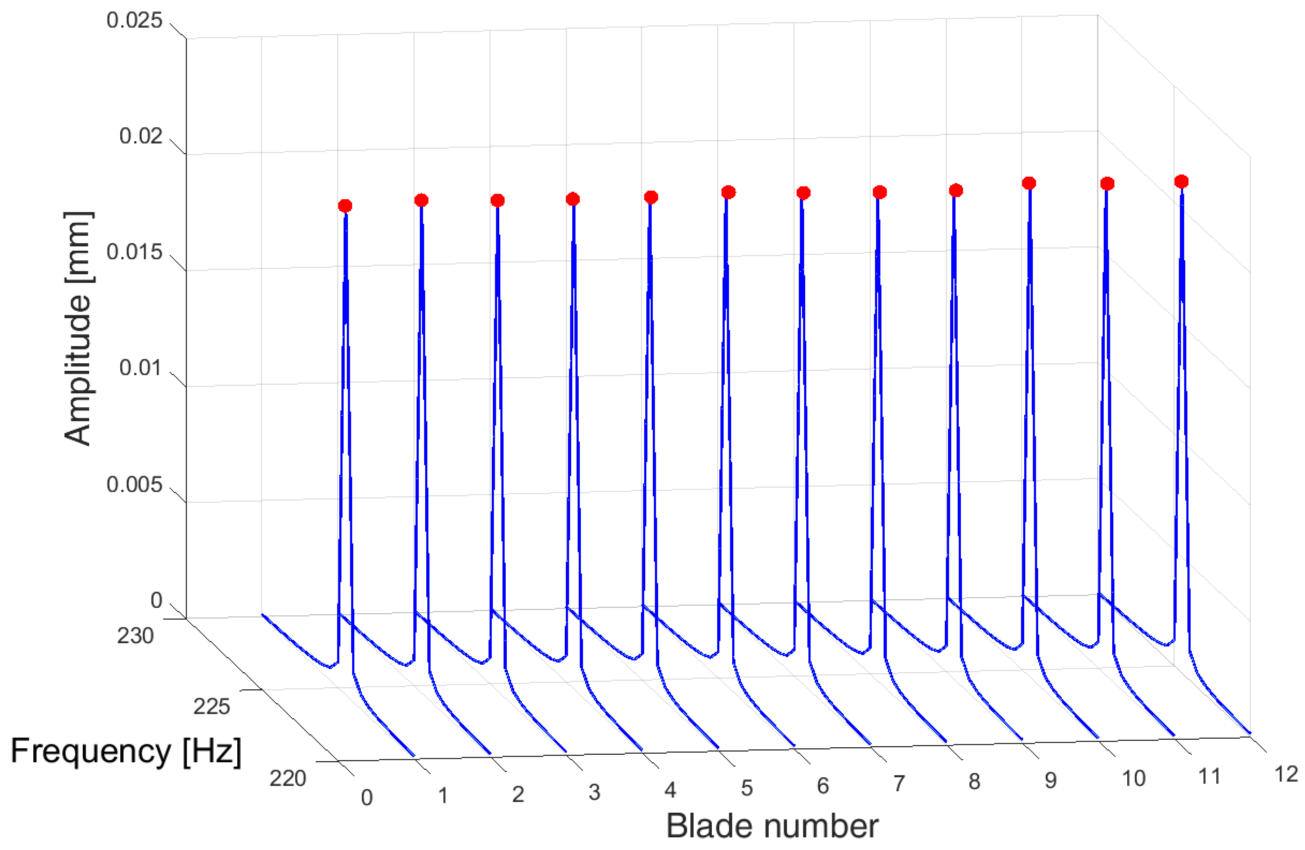




In an ideal perfectly tuned disk it is a straight line

TRAVELLING WAVE EXCITATION on a NON ROTATING DISK

BLADES' RESPONSE IN FREQUENCY

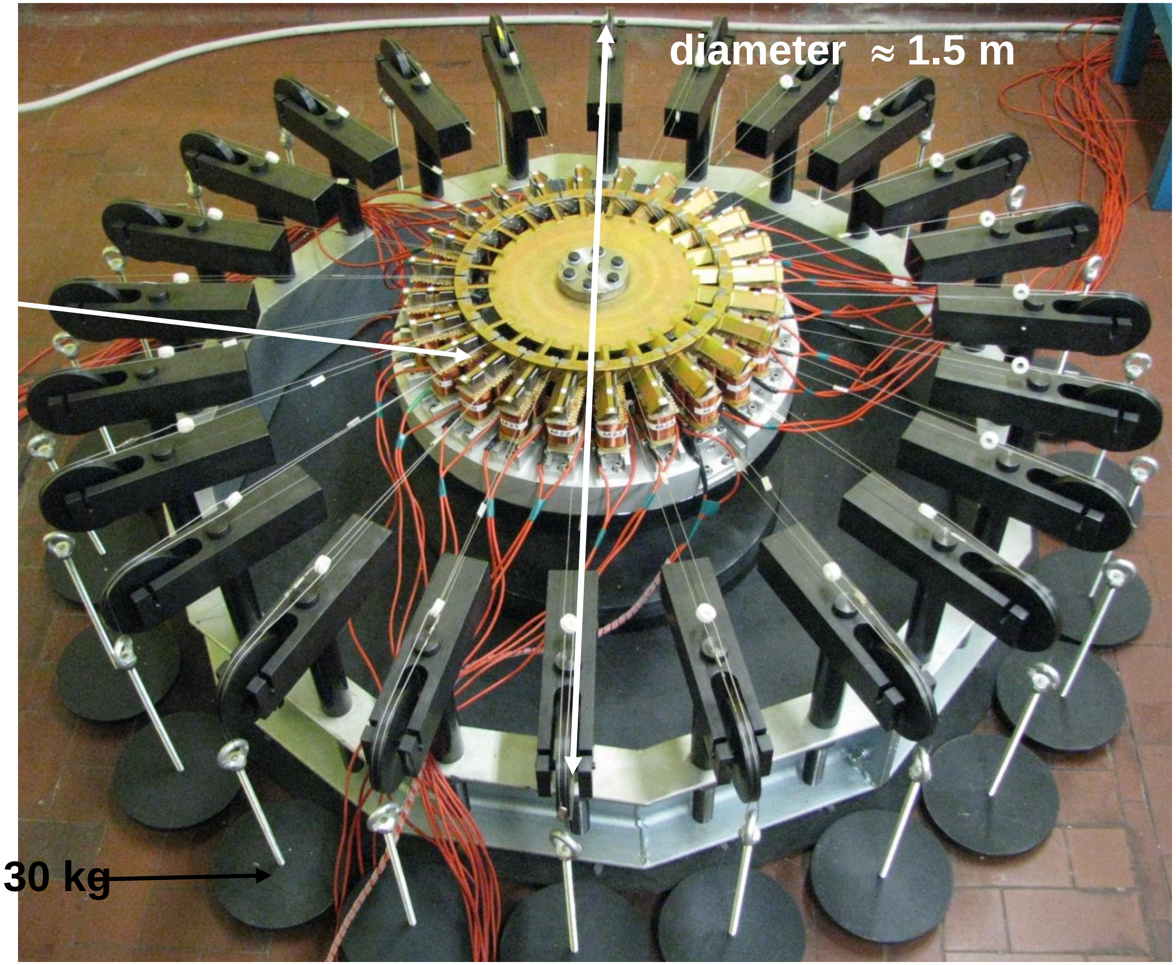


In a tuned disk subjected to a rotating force I expect to find the same response for each blade

The test rigs for bladed disks at Politecnico di Torino

THE TEST RIG OCTOPUS at the Lab. AERMEC Politecnico di Torino

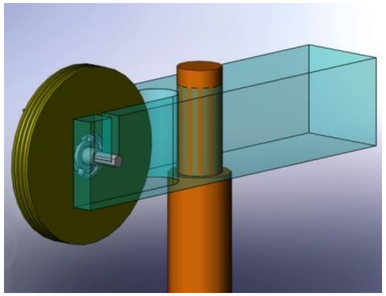
excitation system



load up to 30 kg

Underplatform damper loading system

arm structure



EXCITATION SYSTEM FEATURES

• absence of contact between blades and exciters

•same excitation force amplitude on each blade but different phase

•high excitation force amplitude

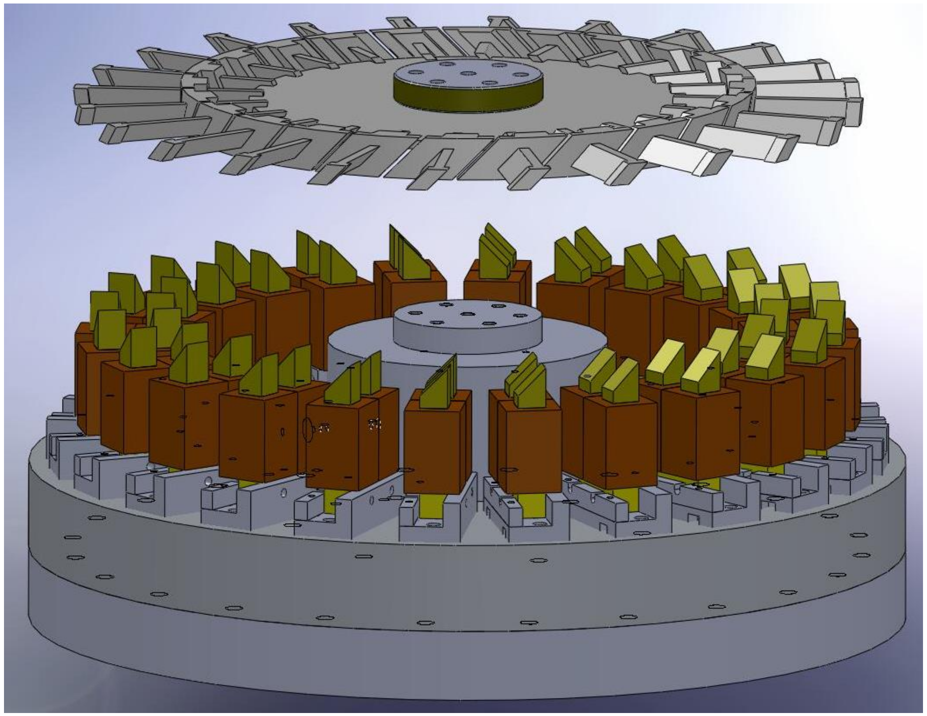
•accurate measurement of the force amplitude

References

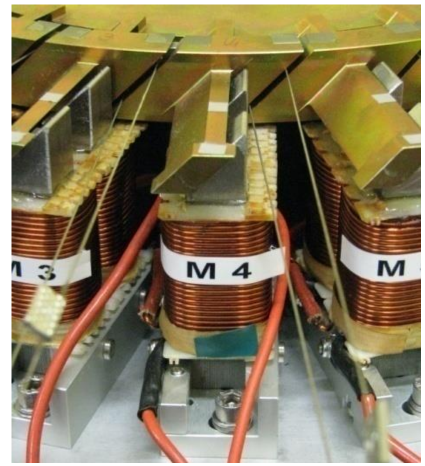
C.M. Firrone, T. Berruti, M.M. Gola, On force control of an engine order type excitation applied to a bladed disk with underplatform dampers, **JOURNAL OF VIBRATION AND ACOUSTICS**, 135(4), 041103 (2013) (9 pages) doi:10.1115/1.4023899.

C.M. Firrone, T. Berruti, - "An electromagnetic system for the non-contact excitation of bladed disks" Experimental Mechanics, DOI: 10.1007/s11340-011-9504-1, Volume 52, Issue 5 (2012), Page 447-459

T. Berruti, C.M. Firrone, M.M. Gola – "A test rig for non-contact travelling wave excitation of a bladed disk with underplatform dampers", Journal of Engineering for Gas Turbines and Power vol 133 Transactions of the ASME, pp. 032502-1-8, ISSN 0742-4795, DOI: 10.1115/1.4002100 (2011).

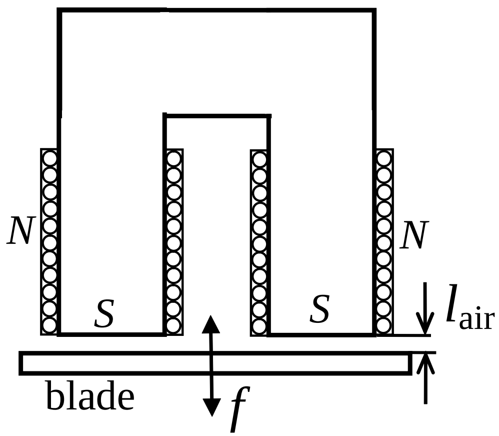


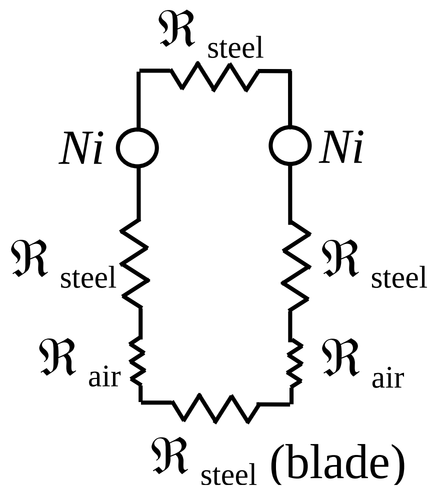
Electromagnets



ferromagnetic extensions

Table I	
Size (b x h x t)	48X64x16 mm
Core section	16x16 mm
turns per coil N	50
Maximum current I_{max}	10 A
Maximum alternating force amplitude F_a	15 N (@ 300Hz 5 N (@ 600Hz

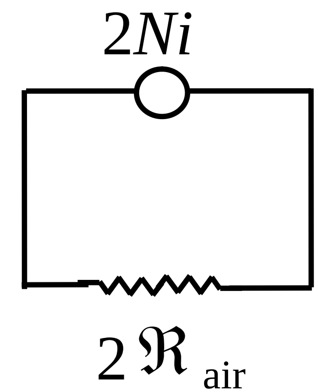




$$\Re = \frac{l}{\mu S}$$

$$\mu_{steel} >> \mu_{air}$$

$$\Re_{steel} << \Re_{air}$$



$$i(t) = I \sin f_{el} t$$

alternate component of the force →

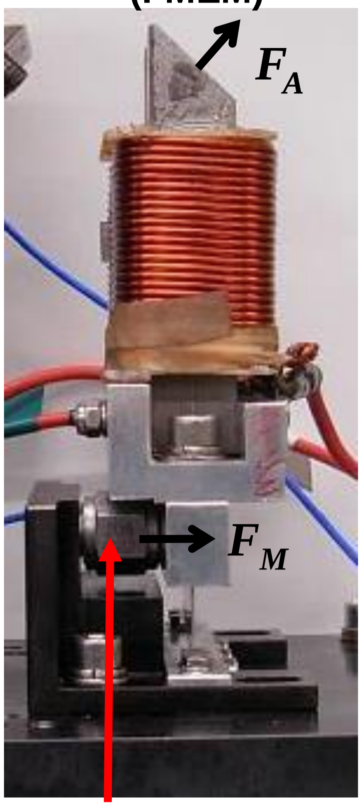
$$f_a = \frac{(N \cdot I)^2 \cdot \mu_{air} \cdot S'}{2 l_{air}^2} \cos 2 f_{el} t$$

$$V = \left(\sqrt{\frac{4 \cdot \pi^2 \cdot F_A \cdot L^2 \cdot l_{air}^2}{\mu_{air} N^2 \cdot S'}} \right) \cdot f_{el}$$

$$V = \left(\sqrt{\frac{4 \cdot \pi^2 \cdot f_{el}^2 \cdot L^2 \cdot l_{air}^2}{\mu_{air} N^2 \cdot S'}} \right) \cdot \sqrt{F_A}$$

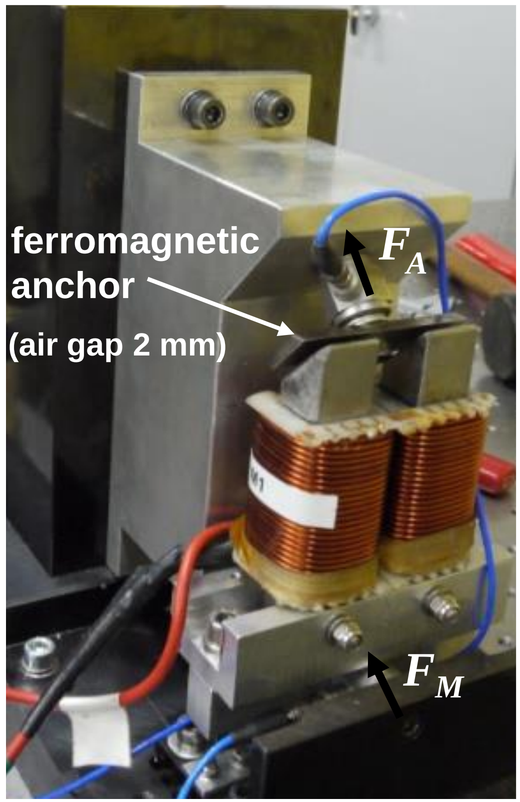
REFERENCE ELECTROMAGNET

The Force Measuring ElectroMagnet (FMEM)

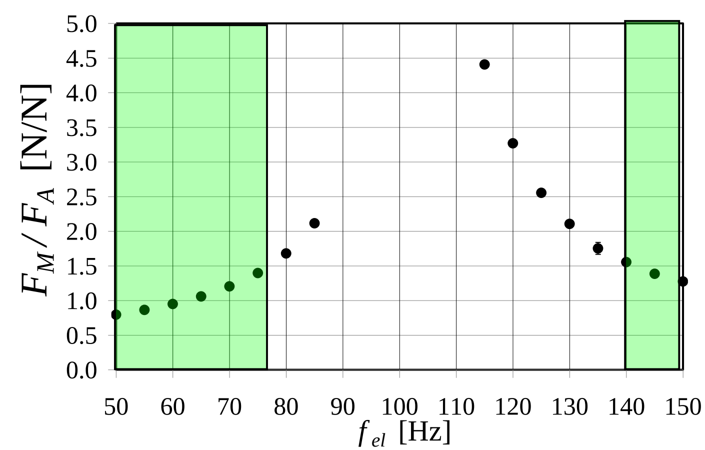


force transducer

FMEM calibration bench

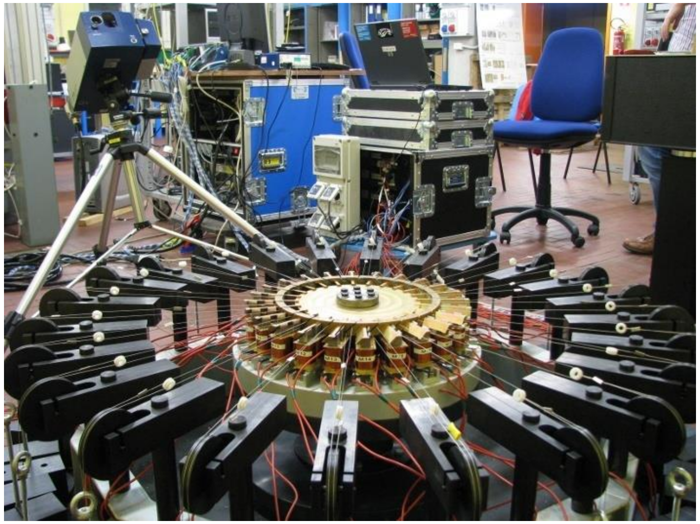


The FMEM calibration curve



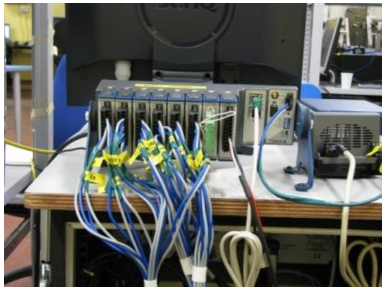
F_A= 5 N (toll. 1%) controlled by a Labview routine

TEST FACILITIES



EMs power supply

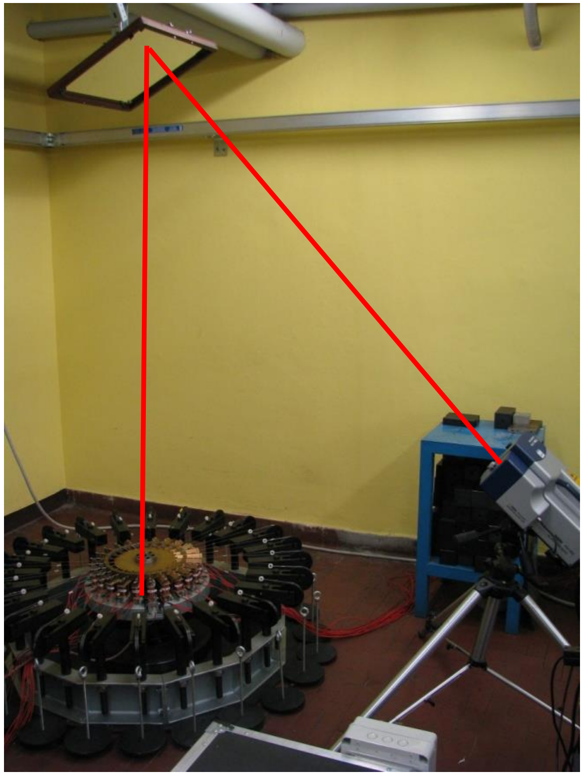
Signal generator
and controller
NI cRIO

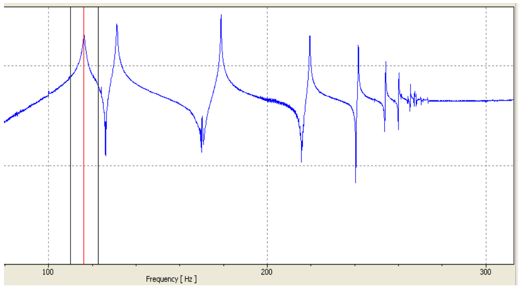


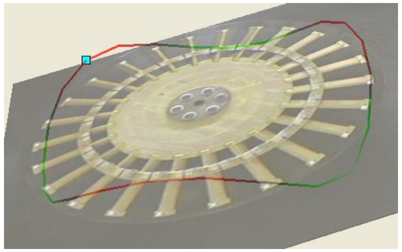
12 amplifiers
2 channel (800 W)

Dynamic response measurement

Laser scanning
vibrometer

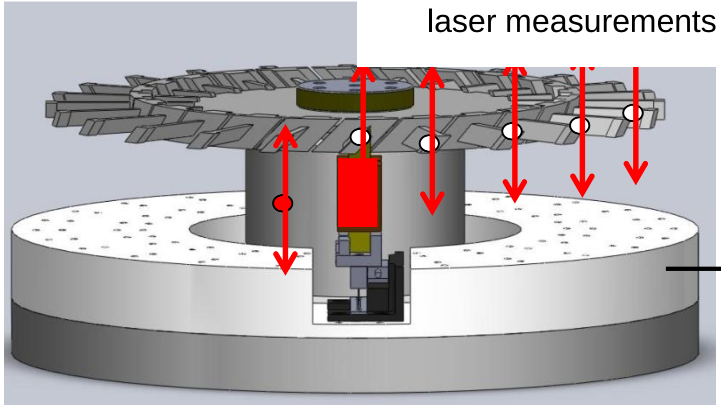


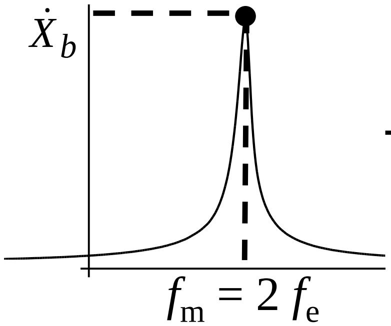


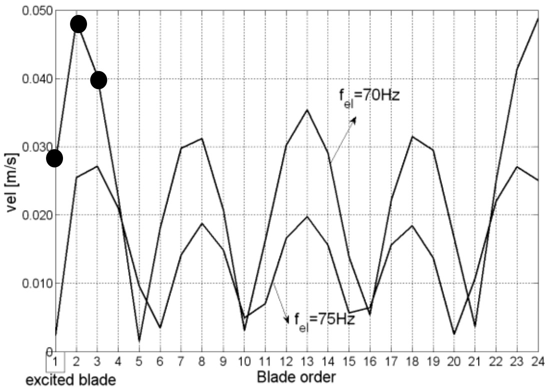


CALIBRATION vs. the REFERENCE ELECTROMAGNET

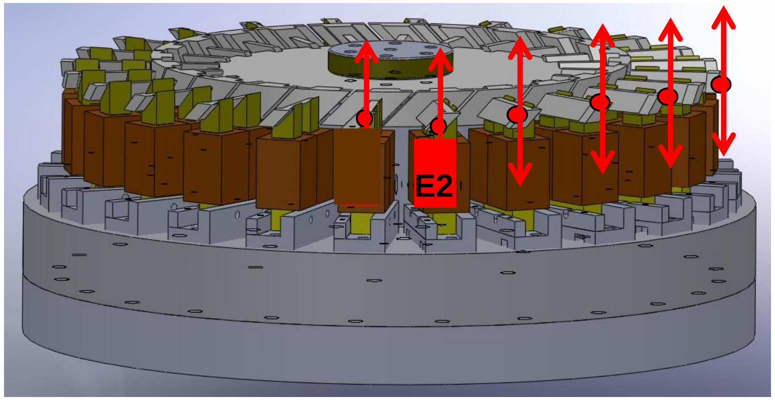
Configuration 1



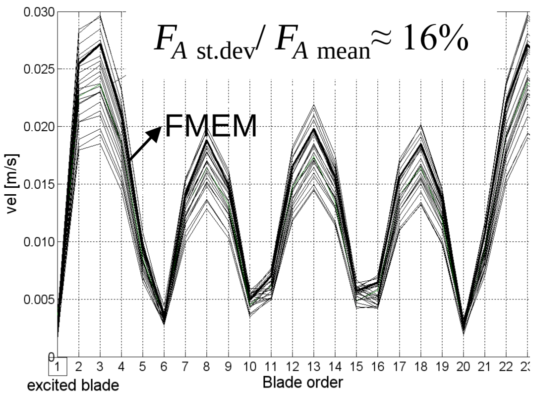




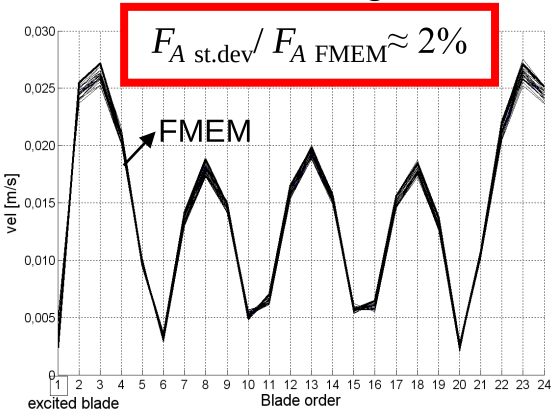
Configuration 2



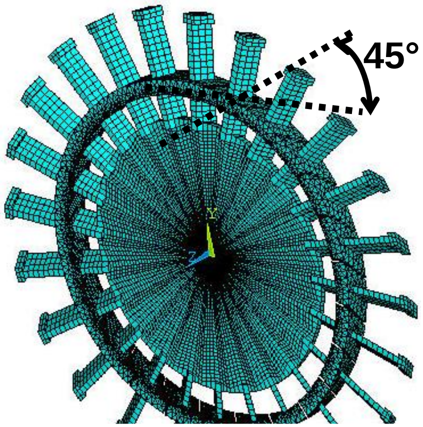
Before tuning

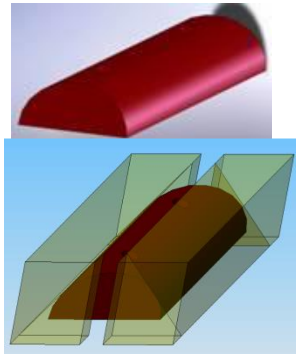


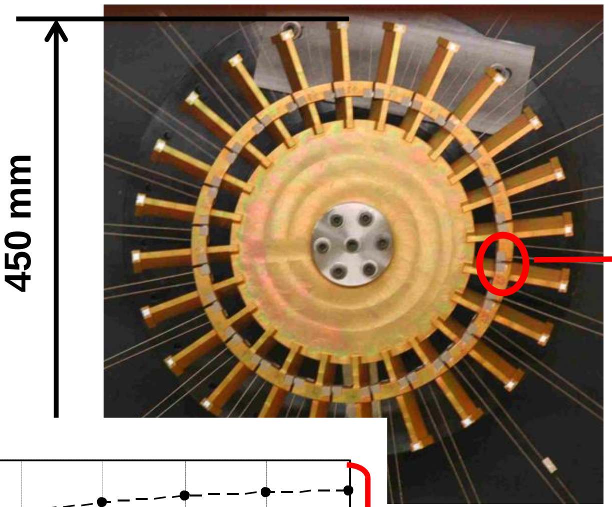
After tuning

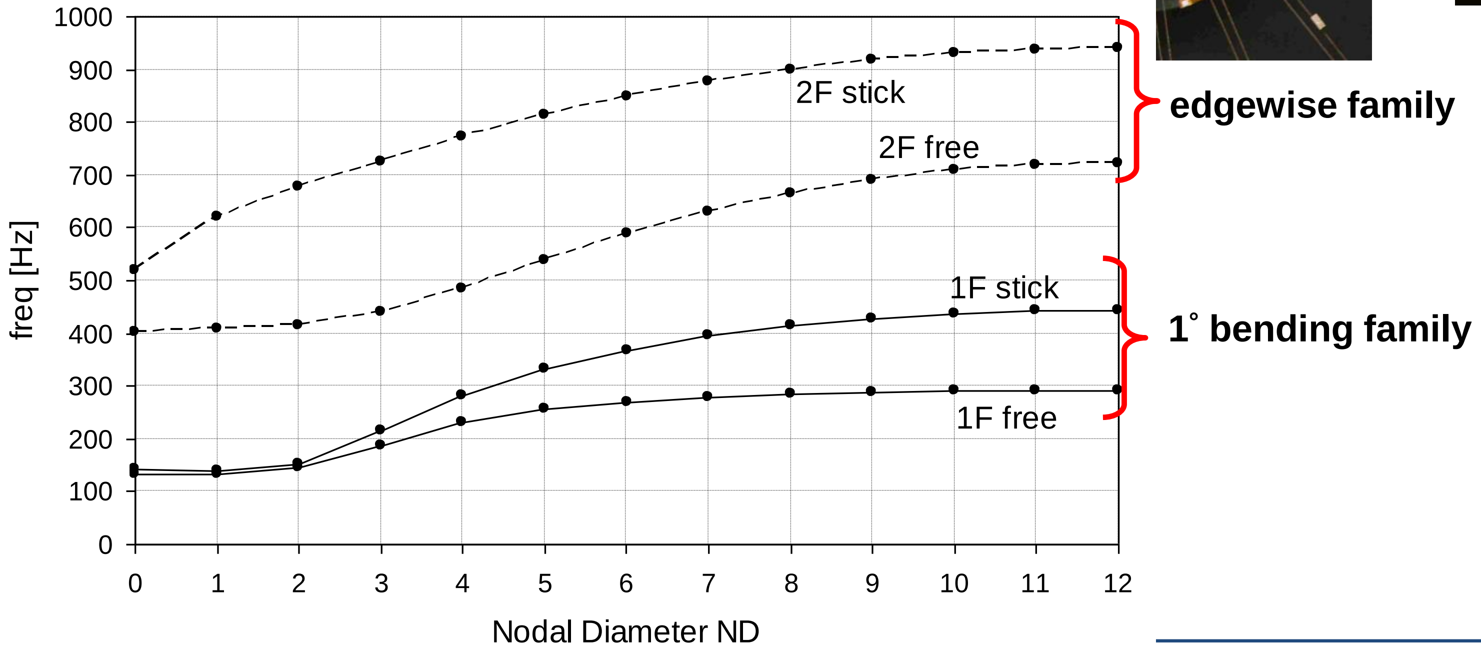


THE BLADED DISK

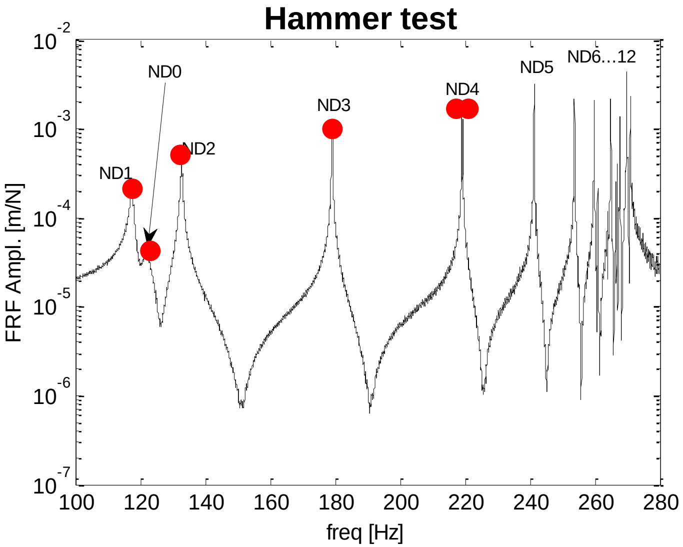




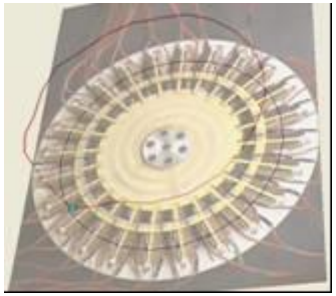




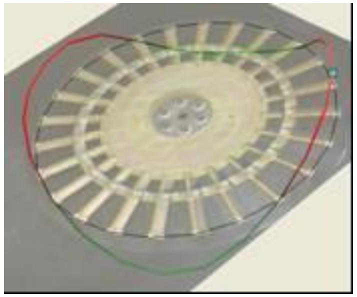
BLISK WITHOUT DAMPERS - HAMMER TEST



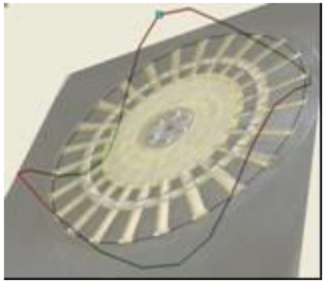
ND = 0



ND = 2

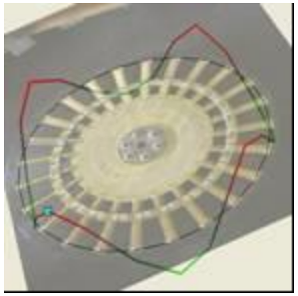


ND = 3

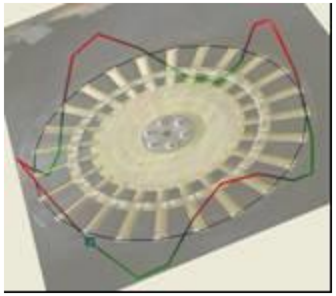


ND = 4

f = 219,1 HZ

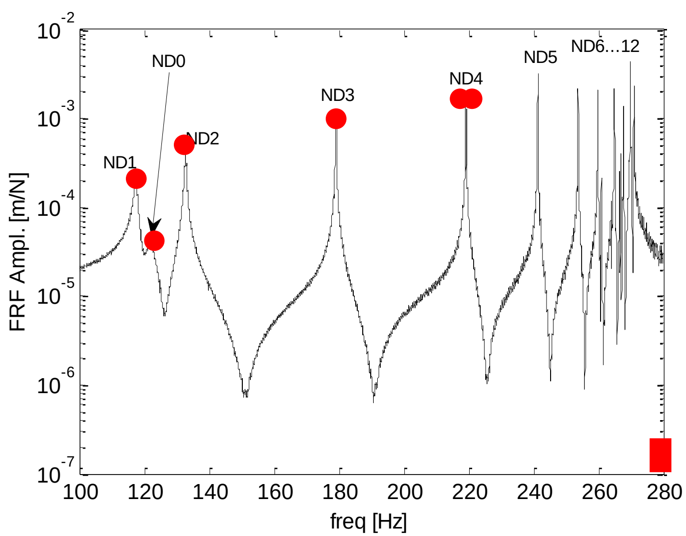


f = 219,2 HZ

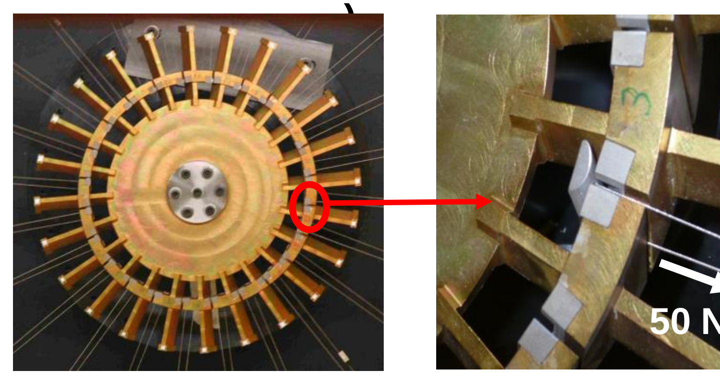


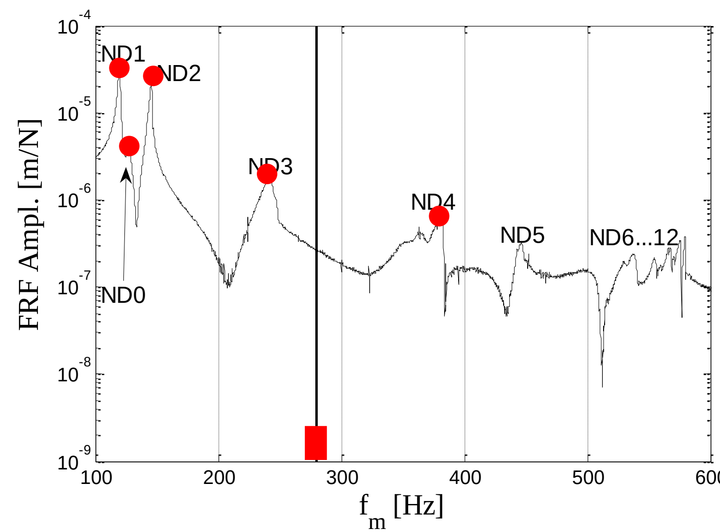
HAMMER TEST – BLISK WITHOUT and WITH DAMPERS

FRF without dampers (free)

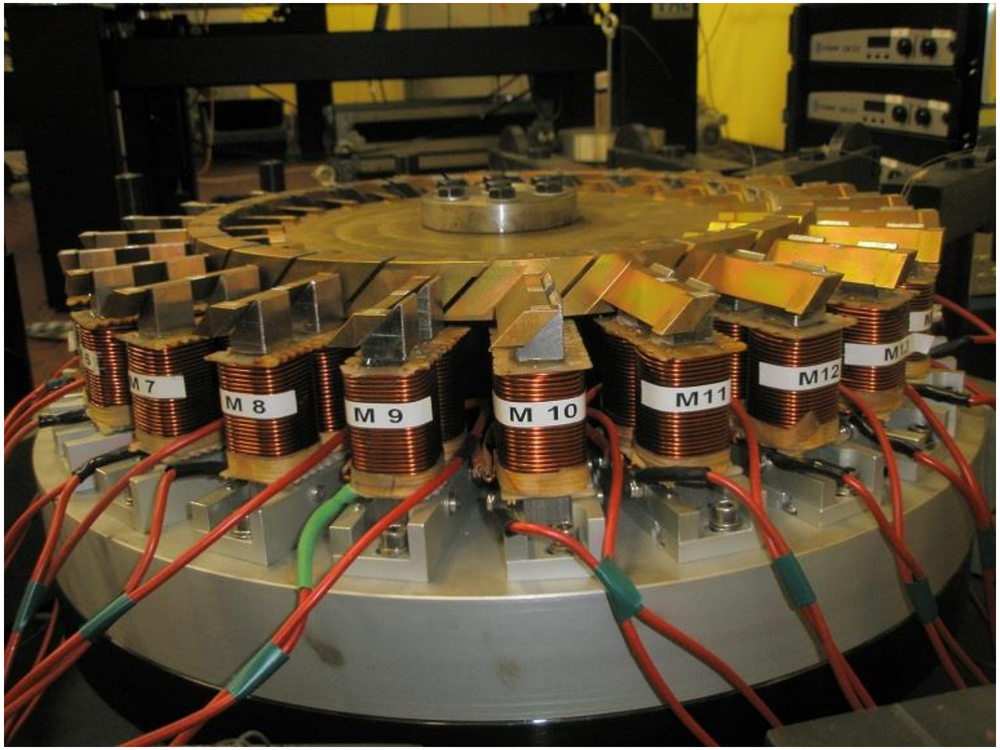


FRF with dampers (50 N on each UPD)





TRAVELLING WAVE EXCITATION



TEST PARAMETERS

$EO = ND = 2,3,4,5,\ 6$

$F_A = 0.1\ \ \mathrm{N}$

Excitation force at a given *EO*

$$f_e = F_E\, cos(\,\omega_m t + (\,n - 1\,)\cdot \varphi_m\,)$$

$n = 1,\ldots,N_b = 24$ number of the blade

EO engine order index

$$\varphi_m = \frac{2\pi}{N_b} EO$$

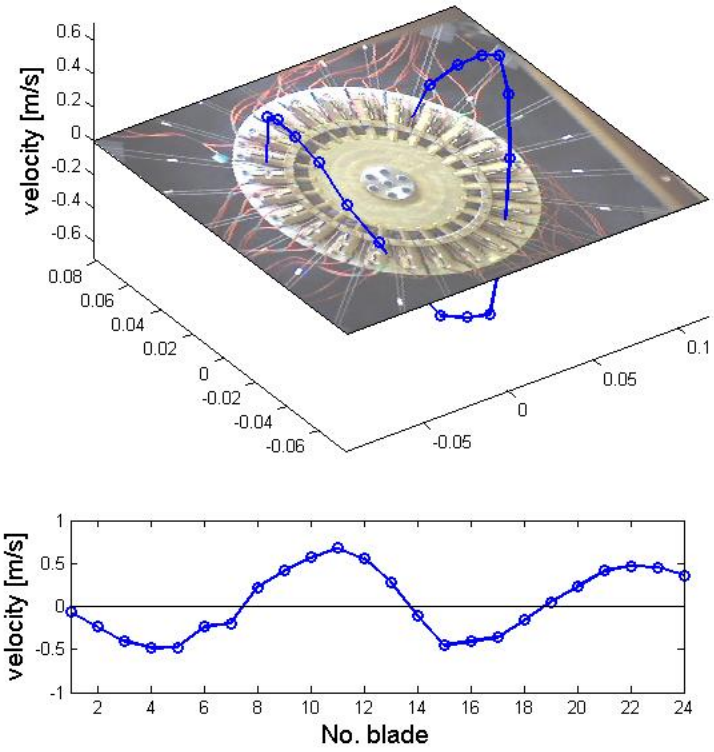
We choose $EO = ND$

$$\varphi_m = \frac{2\pi}{N_b} ND$$

Inter Blade Phase Angle (IBPA)

RESPONSE TO A TRAVELLING WAVE EXCITATION

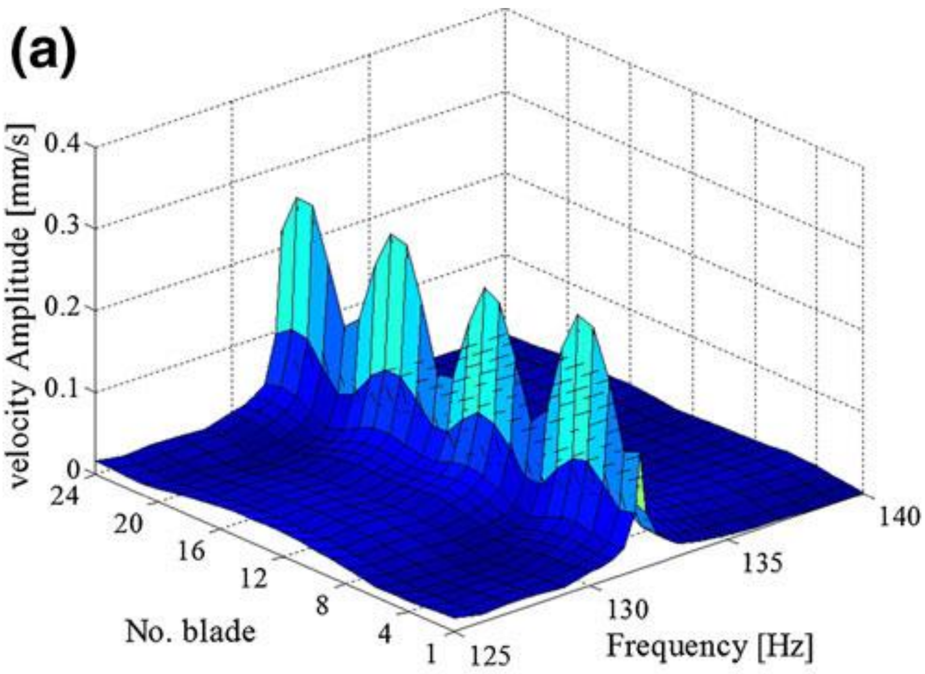
CASE OF SMALL MISTUNING



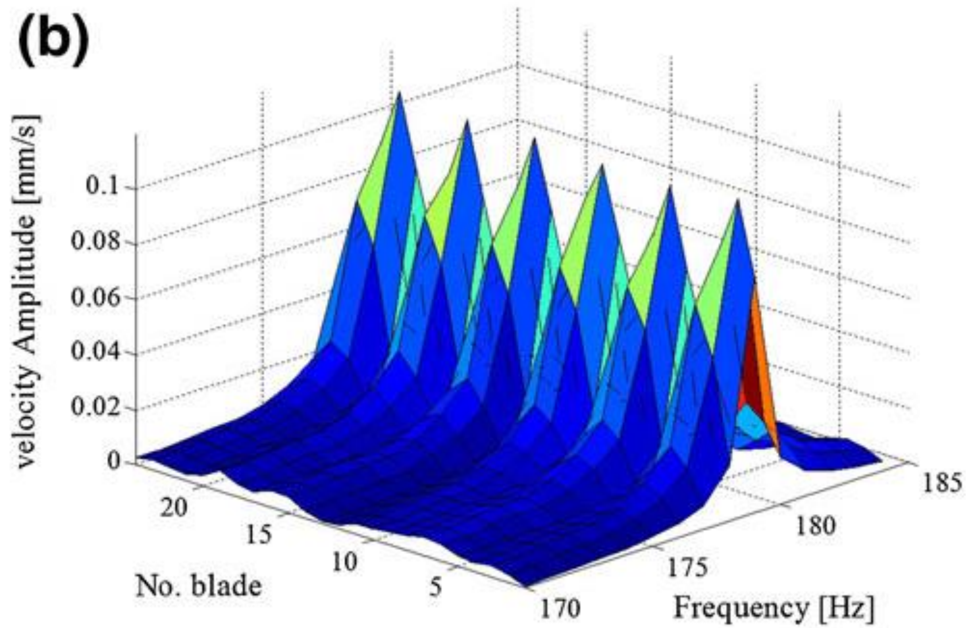
No more a straight line

Small mistuning

Excitation at $EO = ND = 2$



Excitation at $EO = ND = 3$



Why 4 lobes?

Why 6 lobes?

EXPLANATION OF THE MODULATION OF THE RESPONSE

Equation of the travelling response of a tuned disk

$$x = X\,cos(\omega_m t + (n-1)\cdot\varphi_m)\qquad n = 1,\ldots,N_b = 24$$

$$x = X\,cos((n-1)\cdot\varphi_m)\cdot cos(\omega_m t) - X sen((n-1)\cdot\varphi_m)\cdot sen(\omega_m t)$$

$$\varphi_m = \frac{2\pi}{N_b}EO$$

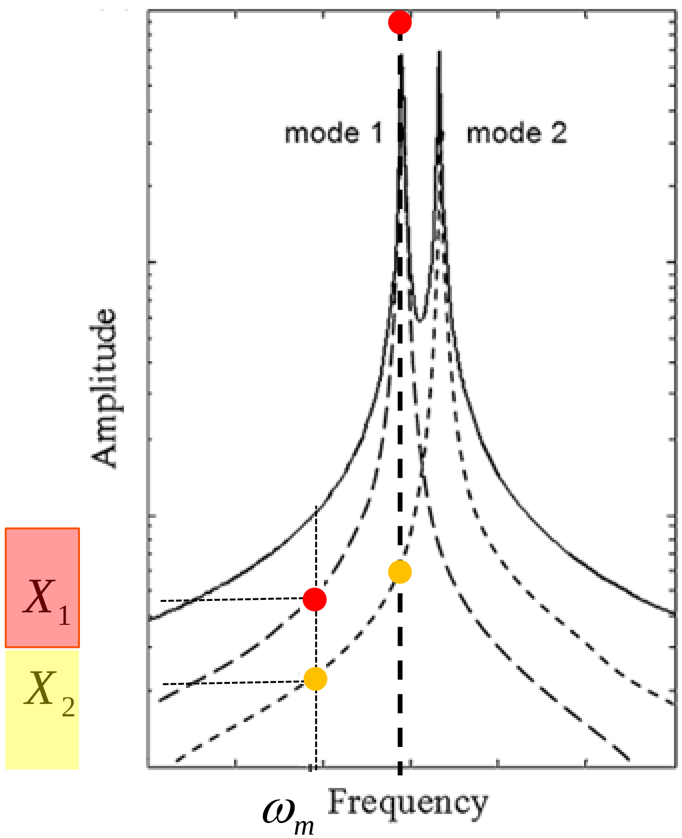
EO engine order index

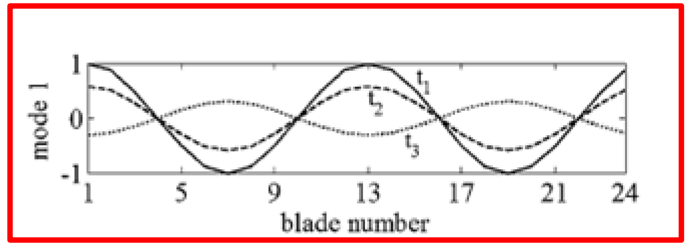
Response of a “small” mistuned disk

$$x = X_1\,cos((n-1)\cdot\varphi_m)\cdot cos(\omega_m t) - X_2 sen((n-1)\cdot\varphi_m)\cdot sen(\omega_m t)$$

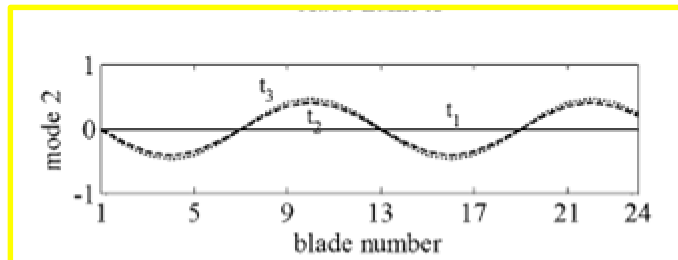
Graphical explanation of the lobes

$$x = X_1 \cos((n-1)\cdot\varphi_m)\cdot\cos(\omega_m t) - X_2\, sen((n-1)\cdot\varphi_m)\cdot sen(\omega_m t)$$

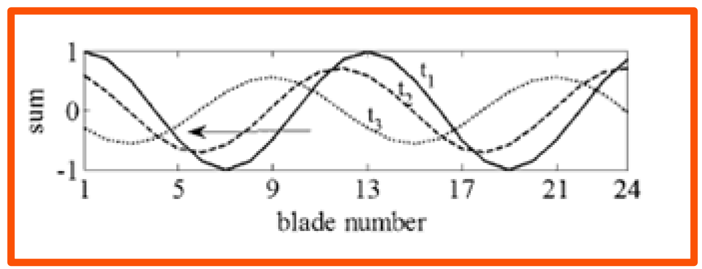




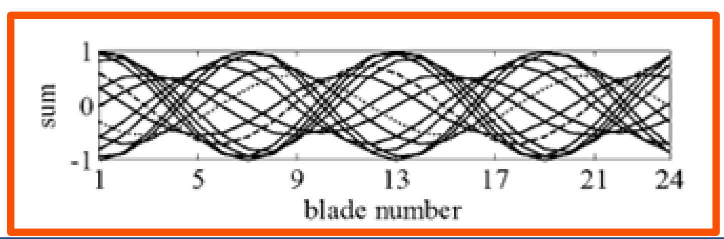
Wave ND2, phase 0°



Wave ND2, phase 90°



Sum of the 2 waves

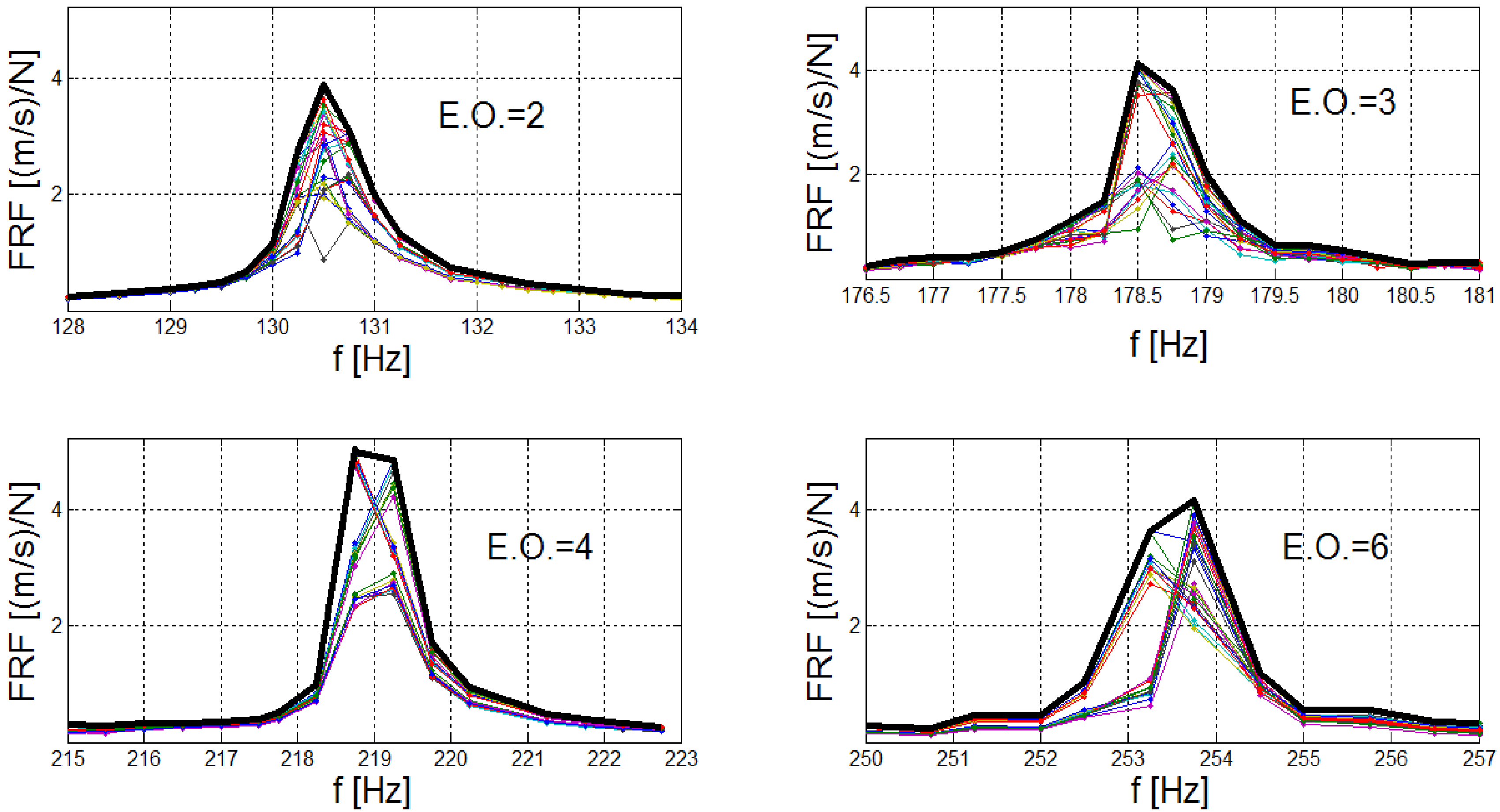


Amplitude modulation

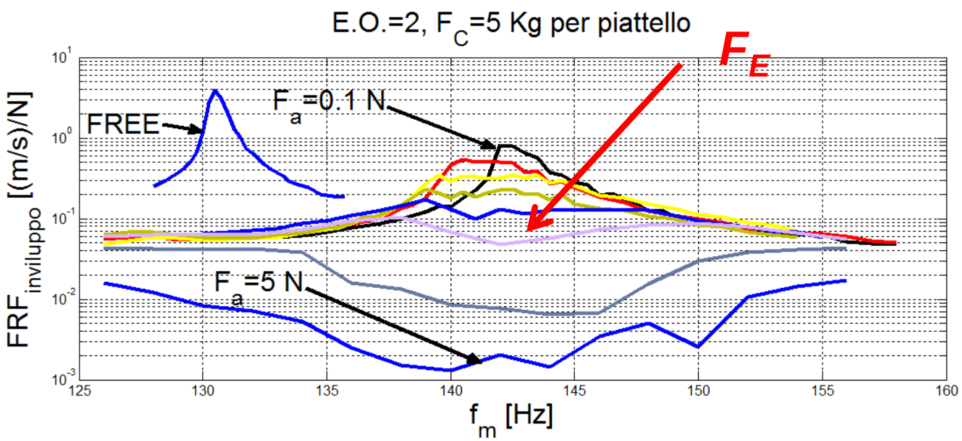
4 LOBES

Envelope curve of the maximum amplitude

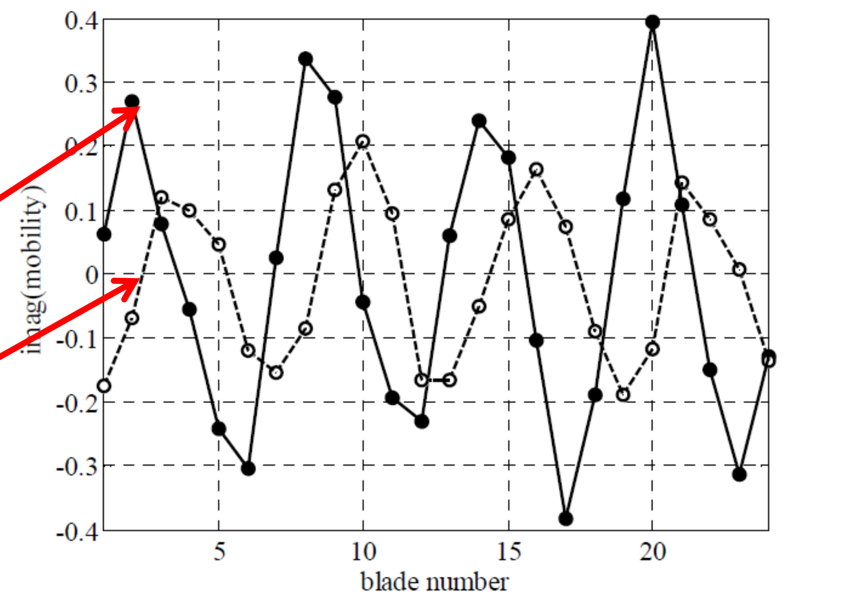
……Let's take the envelope of the amplitudes

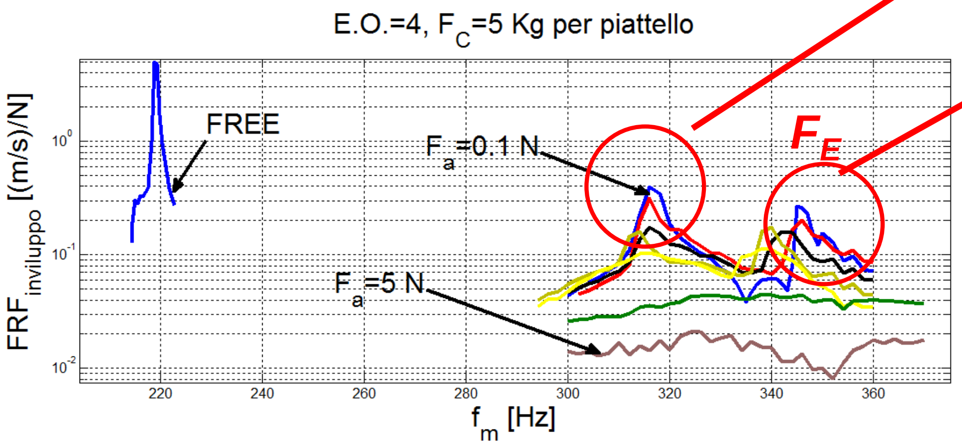


Response for different excitation force amplitudes



Operative deformed shapes of the two modes





Standing waves…no more rotating

Split of frequency due to mistuning

Forced response measurement on rotating bladed disks

Berruti T.M., Firrone C.M., Gola M.M., Calza P., "The effect of friction contacts on the dynamics of a rotating vane segments array : simulation and comparison with experimental result, World Tribology Congress, Torino 2013, pp1-4, ISBN 9788890818509.

Berruti T.M, Maschio V. –" Experimental investigation on the forced response of a dummy counter rotating turbine stage with friction damping", J of Eng. For gas turbines and power, vol. 134 n. 12. - ISSN 0742-4795 (2012) doi: 10.1115/1.40

Battiato, G, Firrone, C.M.; Berruti, T. M. (2016) Forced response of rotating bladed disks: Blade Tip-Timing measureaments, Mechanical Systems and Signal Processing vol. 85, pp. 912-926.

Rigosi G., Battiato G., Berruti T.M., (2017) Synchronous vibration parameters identification by tip timing measurements, Mechanics Research Communications, Volume 79,pp. 7-14.

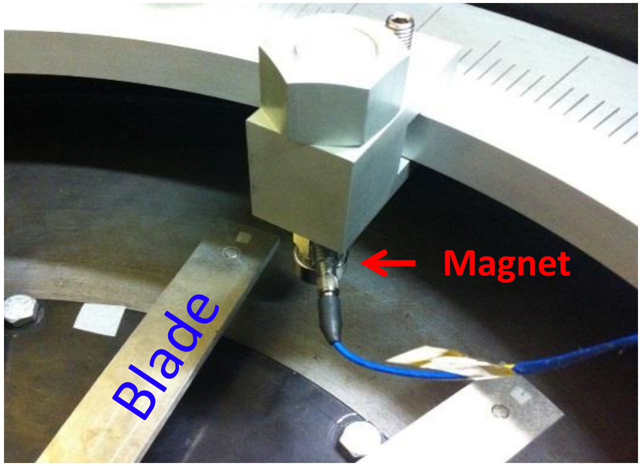
SPINNING TEST RIG

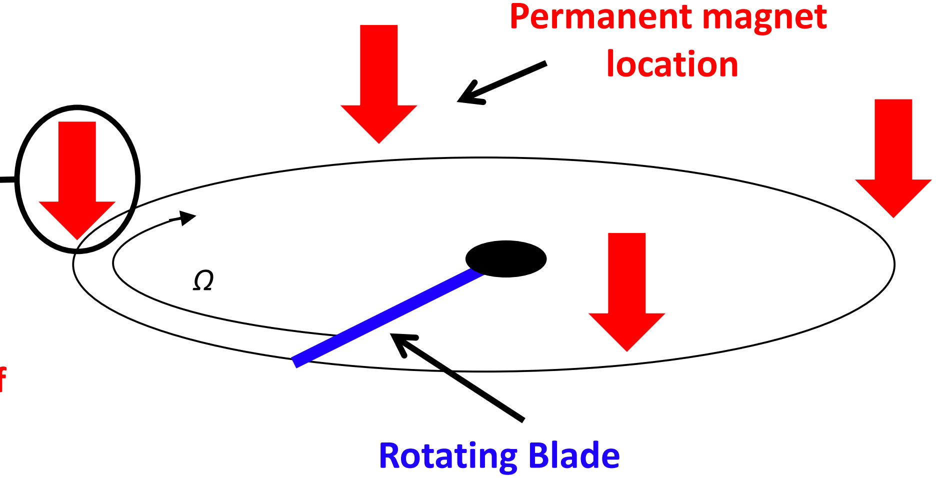
Main features

- rotation speed up to 4000 rpm (2500 rpm with this test article)
- test article diameter up to 650 mm
- 24 channel telemetry system
- permanent magnet excitation (1 to 24 magnets)
- 6 magnets with force transducers

The Excitation System: multiple EOs excitation

The excitation system uses cylindrical permanent magnets facing the rotating blades





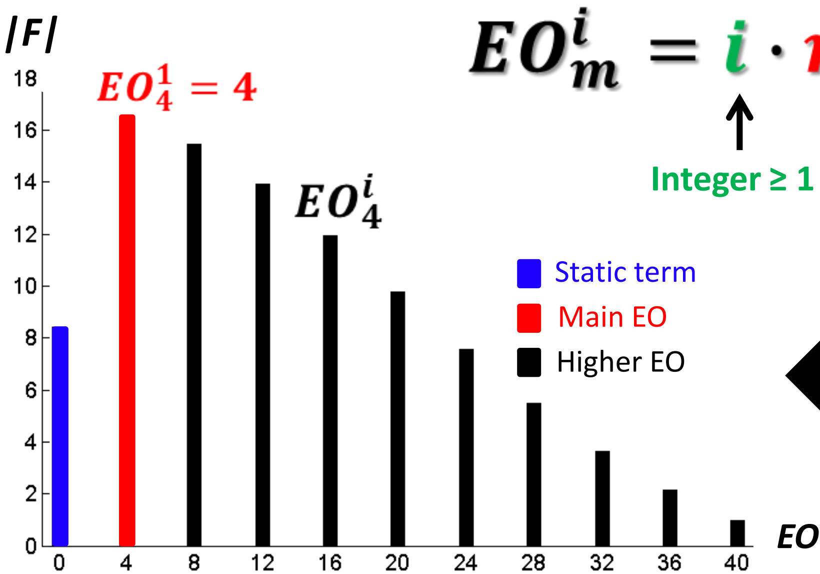
Number of magnets

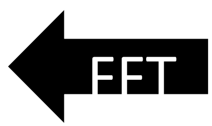
$$EO_m^i = i \cdot m$$

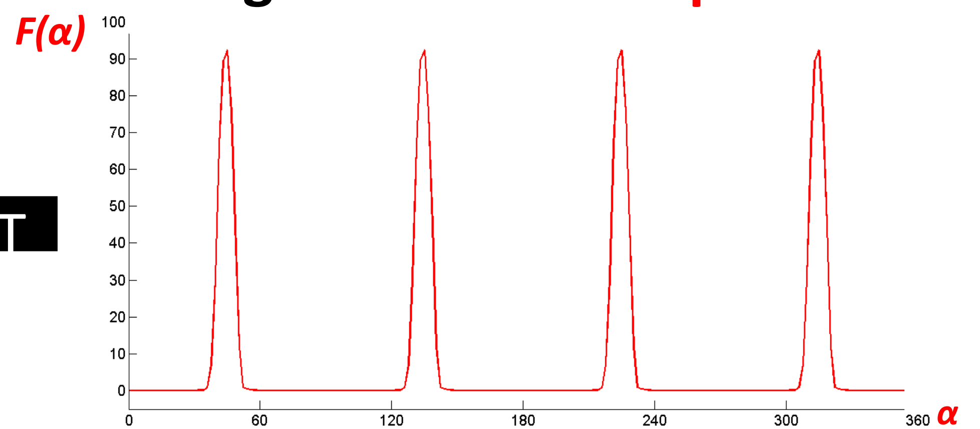
Integer ≥ 1

During the rotation each blade is excited by a number of impulses equal to the number of permanent magnets

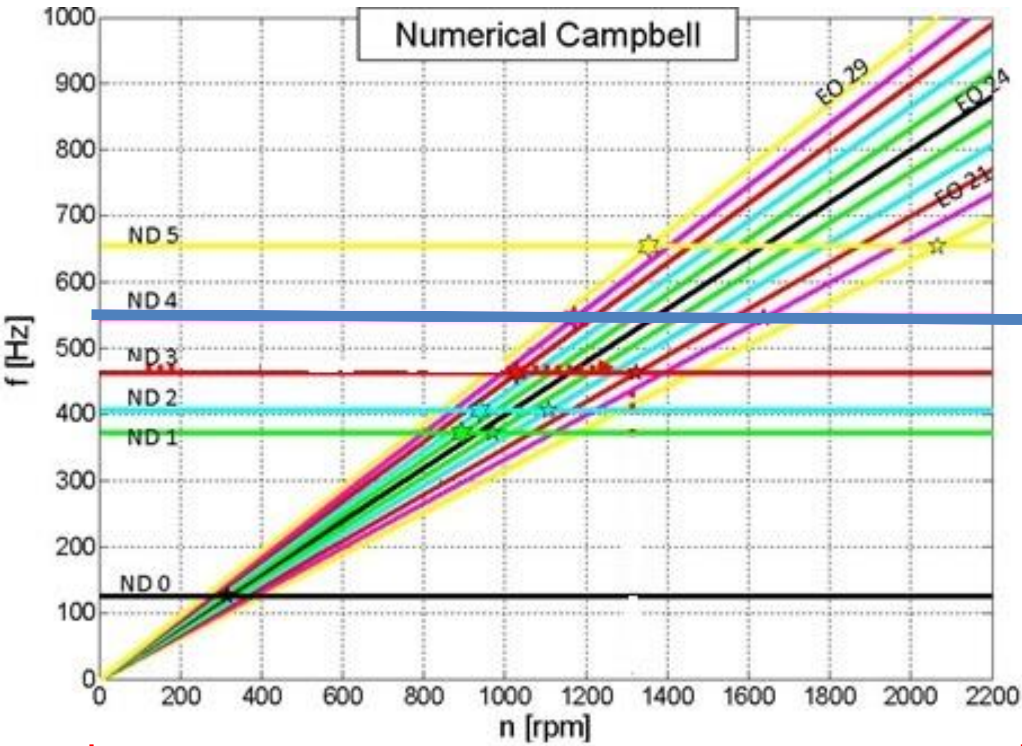
4 magnets → 4 impulses







Choosing the mode to be excited

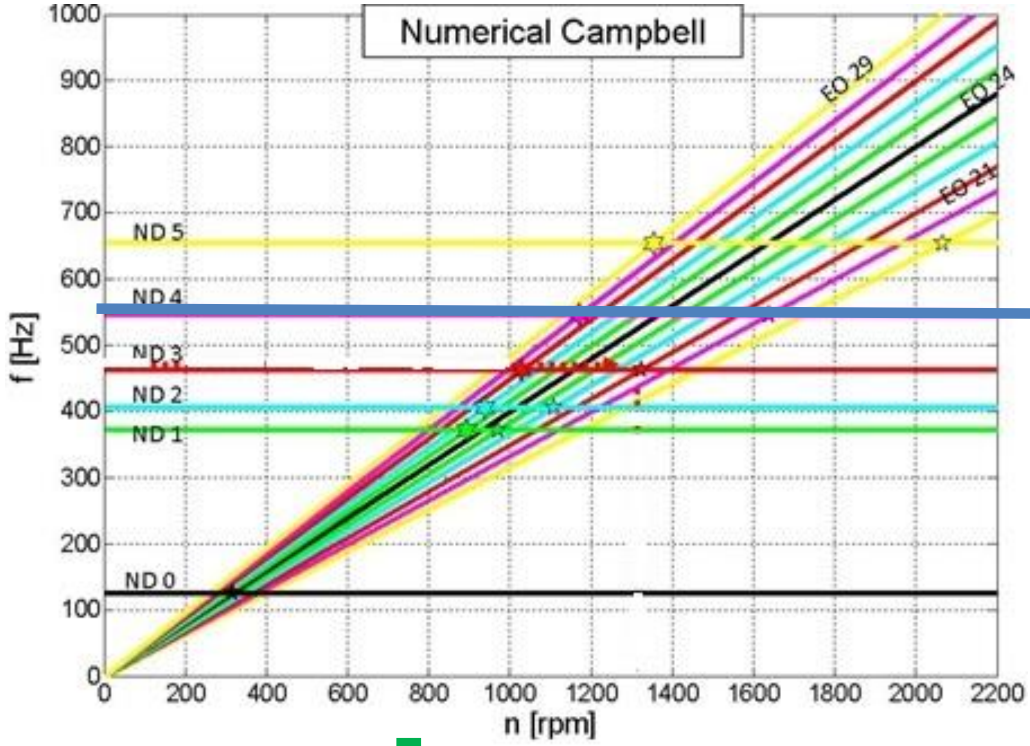


ND=4 f_{res}=552 Hz

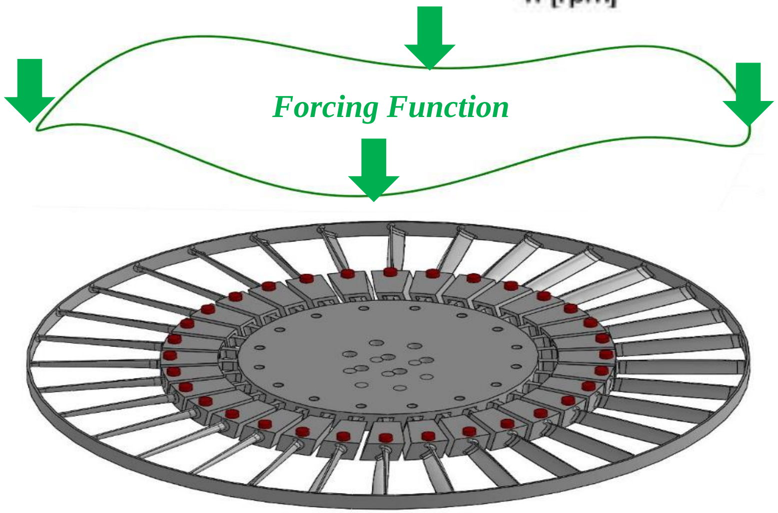
N_b=24 number of blades

Speed range

Choosing the mode to be excited



ND=4 f_{res}=552 Hz

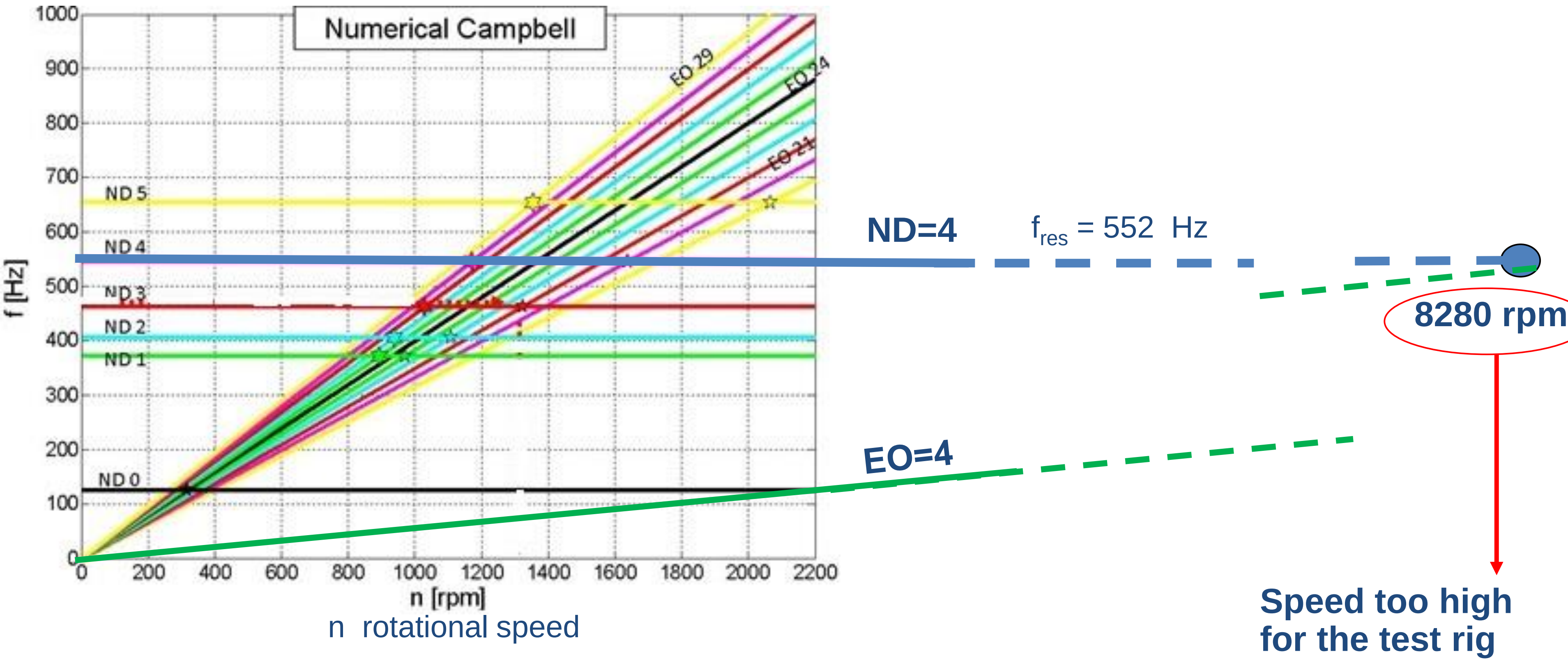


To excite the mode at ND=4…

….we want to meet the two conditions:

$$\begin{cases} f_{exc} = f_{res} \\ EO = ND \end{cases}$$

Choosing the rotation speed



n rotational speed

$$EO \cdot n = f_{res}$$

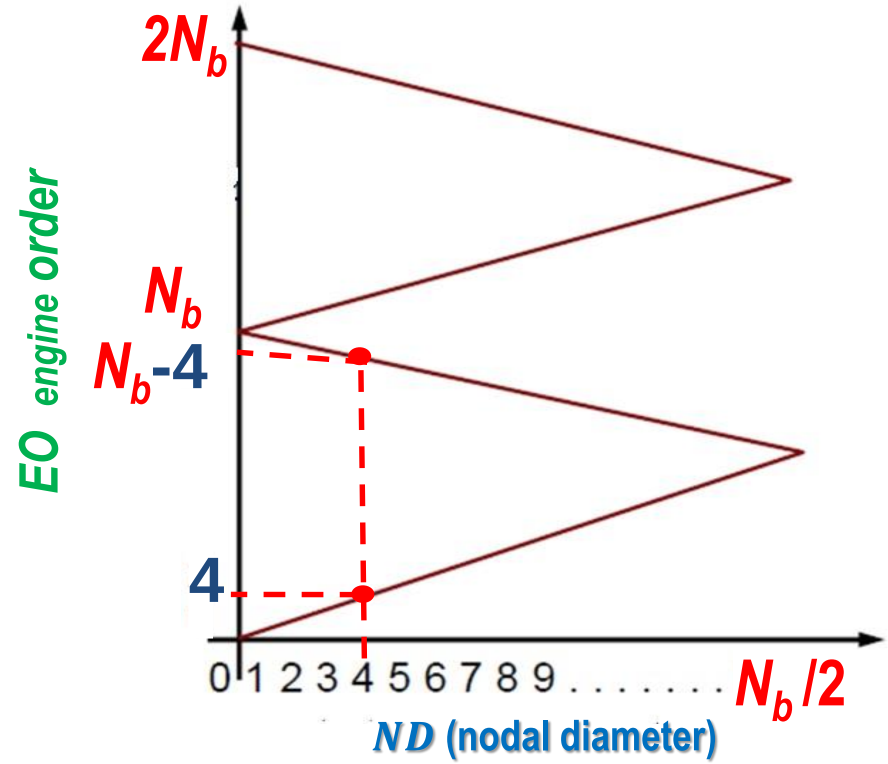
f_{res}= 552 Hz

EO =4 (magnets)

$$n = \frac{60 \cdot f_{res}}{EO} = \frac{60 \cdot 552}{4} = 8280 \text{ rpm}$$

Choosing the Engine Order

Let's use the SAFE diagram…



In the spin rig EO=number of magnets N=24 number of blades

To excite ND=4

EO=4 magnets

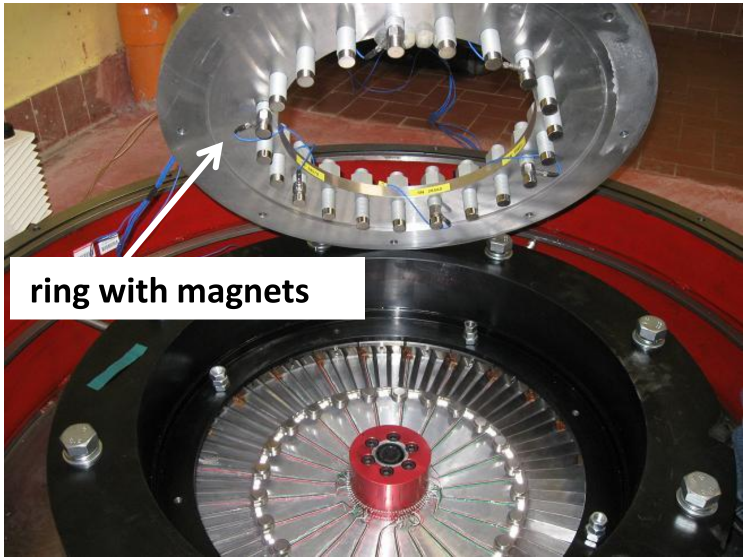
$$n = \frac{60 \cdot f_{exc}}{EO} = \frac{60 \cdot 552}{4} = 8280 \, rpm$$

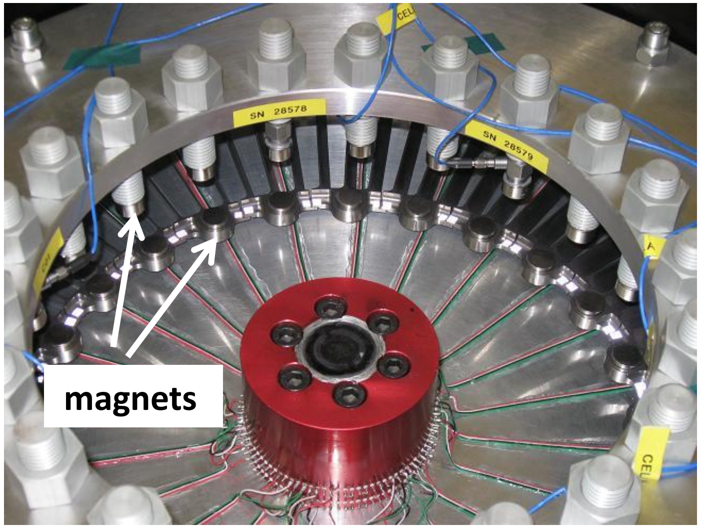
EO=24-4=20 magnets

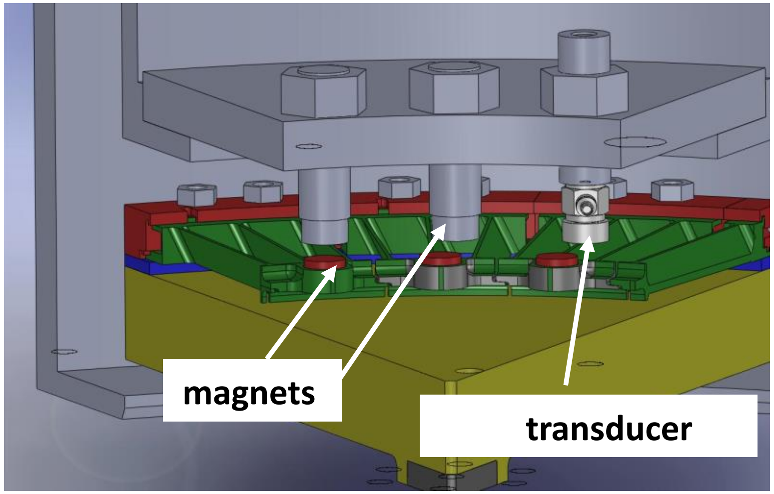
$$n = \frac{60 \cdot f_{exc}}{EO} = \frac{60 \cdot 552}{20} = 1656 \, rpm$$

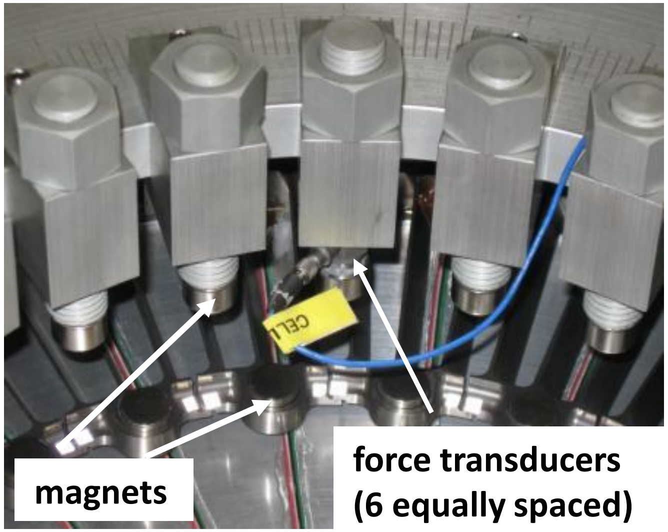
The excitation system: permanent magnet

(example: magnets with diameter 18 mm, height 10 mm, magnetization N52).

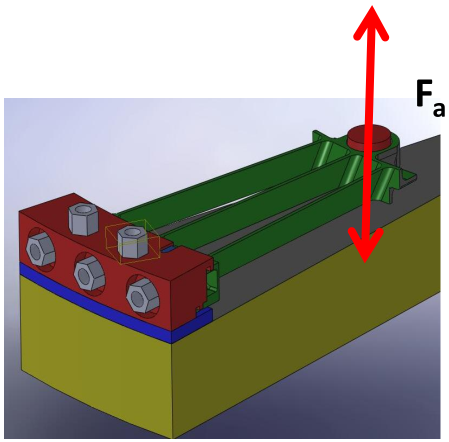


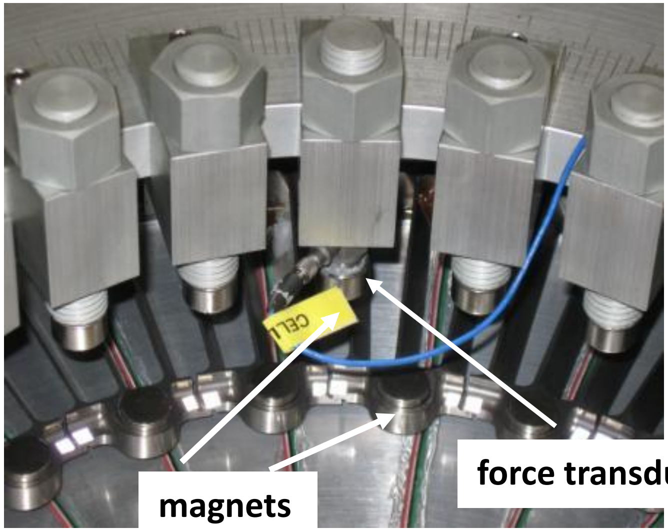






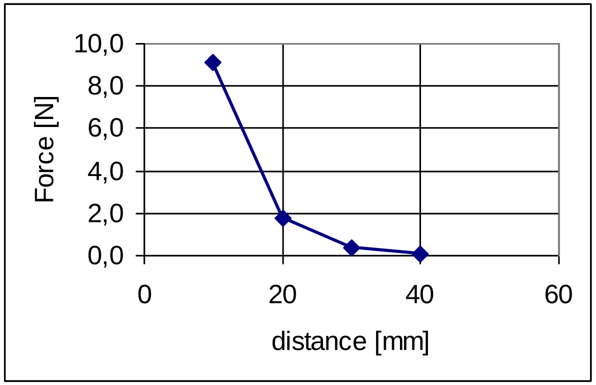
Determination of the magnetic forces





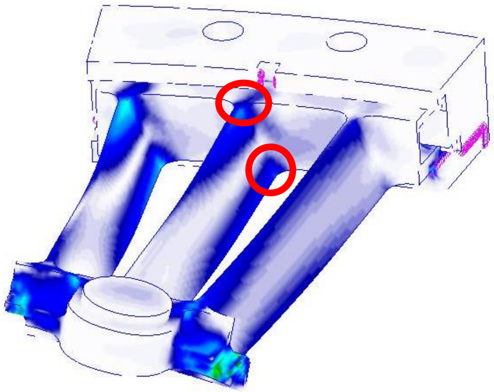
-F_a from measurement of the force transducers.

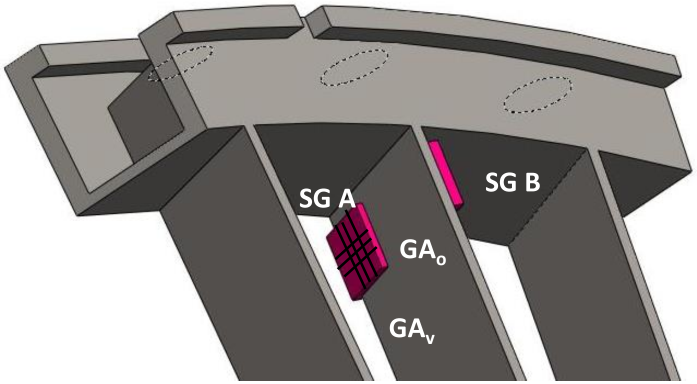
gap [mm]	measured axial force amplitude F_a [N]
10	9.11
20	1.75
30	0.38
40	0.08

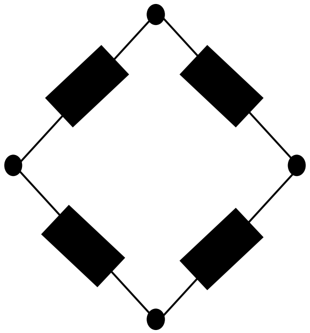


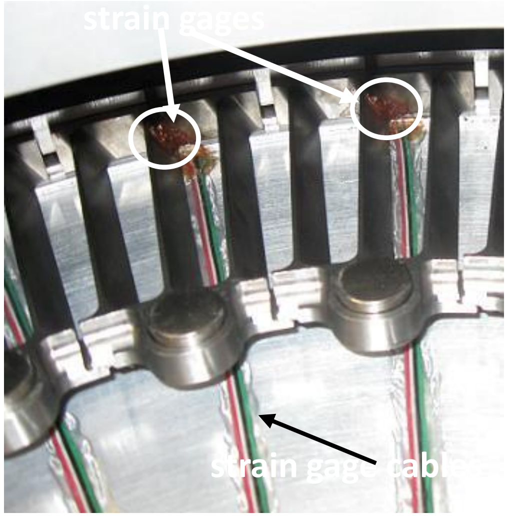
By increasing the gap of 4 times the force decreases of 100 times!

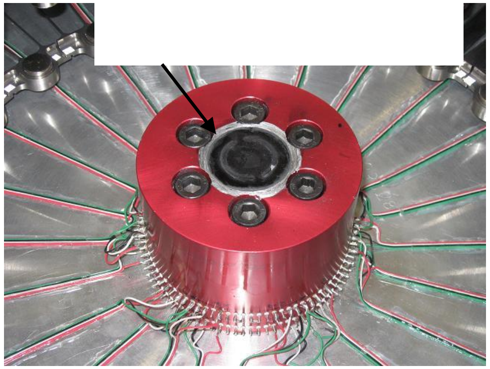
Strain gages and telemetry

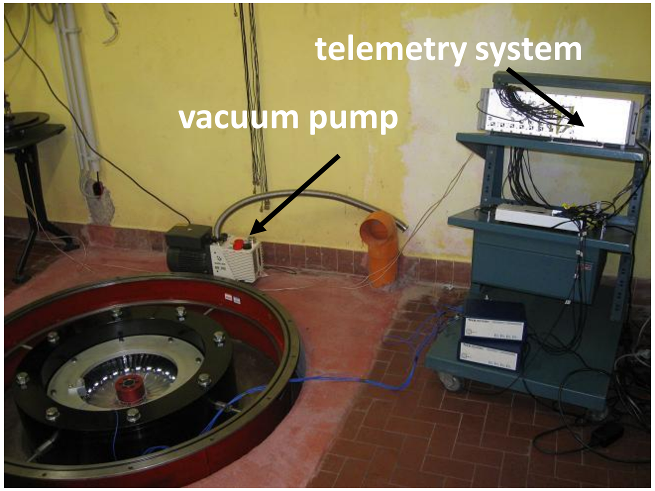




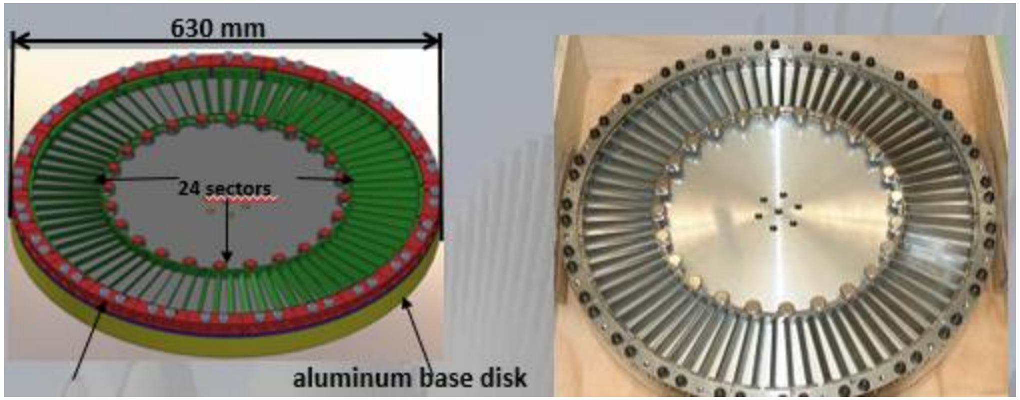








Measurement example



Example

n = 0 ÷ 1500 rpm with Δn = 2 rpm

T = 4 s time acquisition for each step of n

FFT of the signal from each strain gage and store of the amplitude corresponding to the excitation frequency

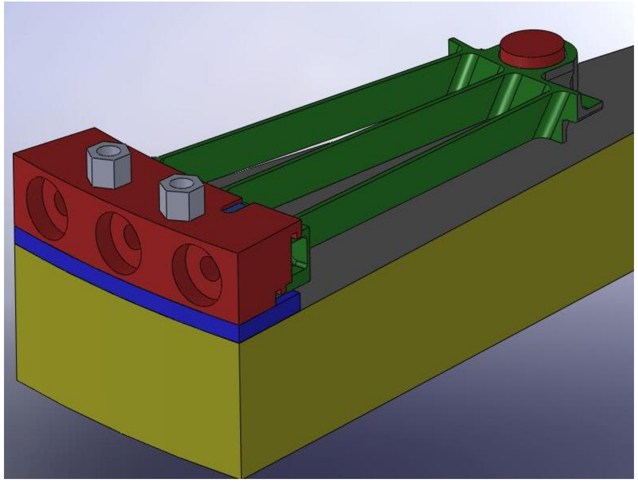
FFT of the signal from each force transducer and store of the amplitude corresponding to the excitation frequency

Calculation of the FRF for each sector;
ratio of the FFT from the strain gage and the FFT of the force (average of the FFTs of the six force transducers)

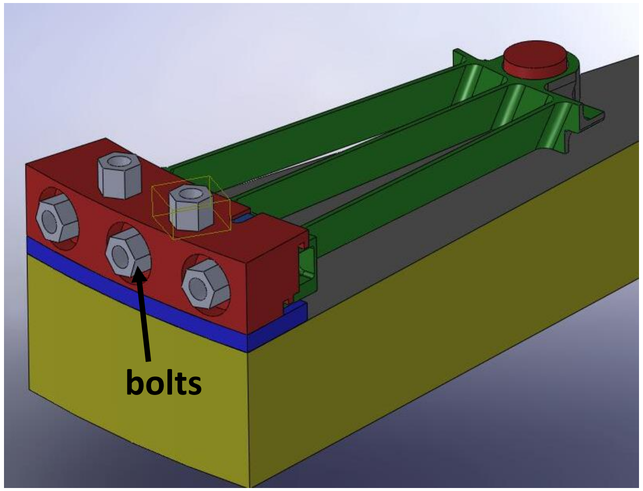
Test configurations

A) sectors connected to the casing through the hooks as in the real case
B) sectors rigidly connected to the casing through bolted joints

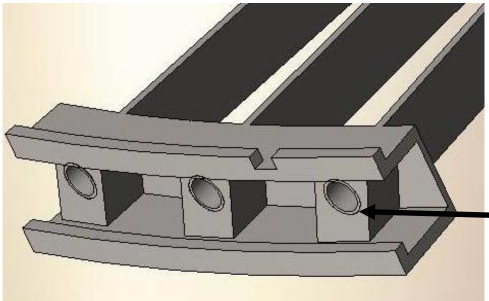
Configuration A
Free hooks



Configuration B
Tightened hooks



attachment detail

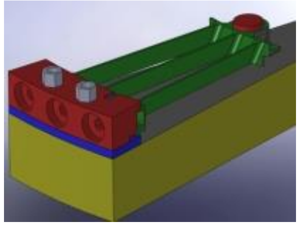


threaded holes

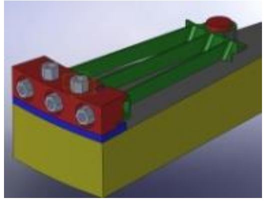
56

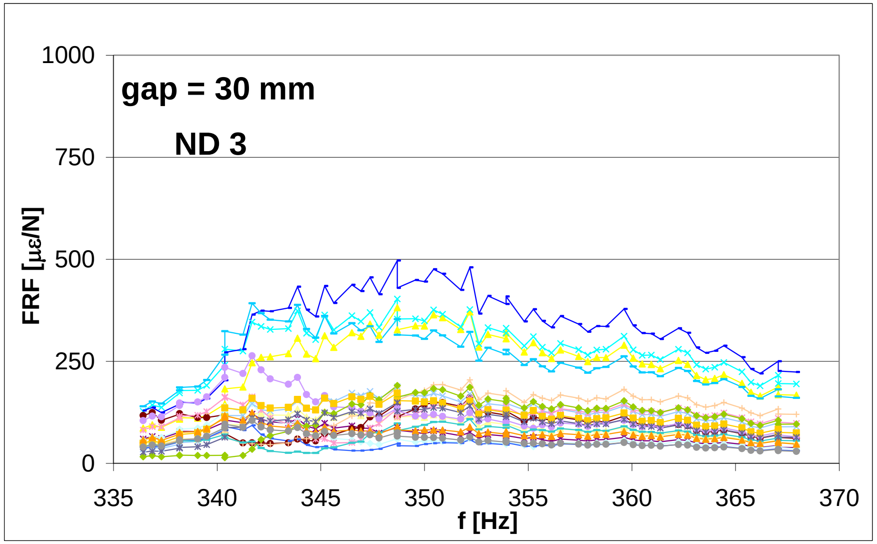
Example of test results

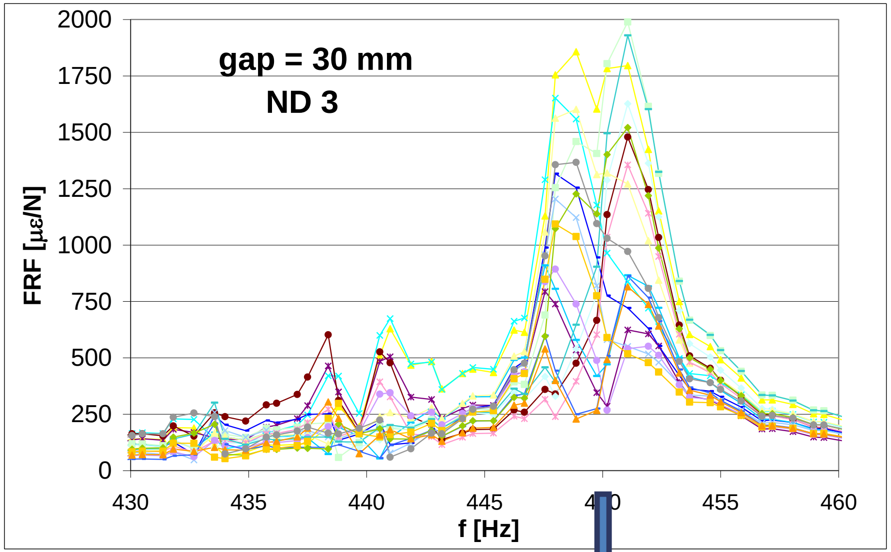
Configuration A - Free hooks



Configuration B - Tightened hooks



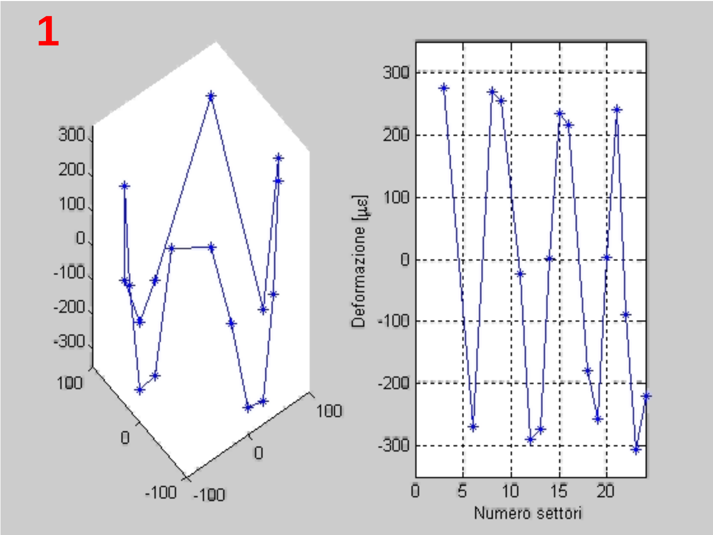


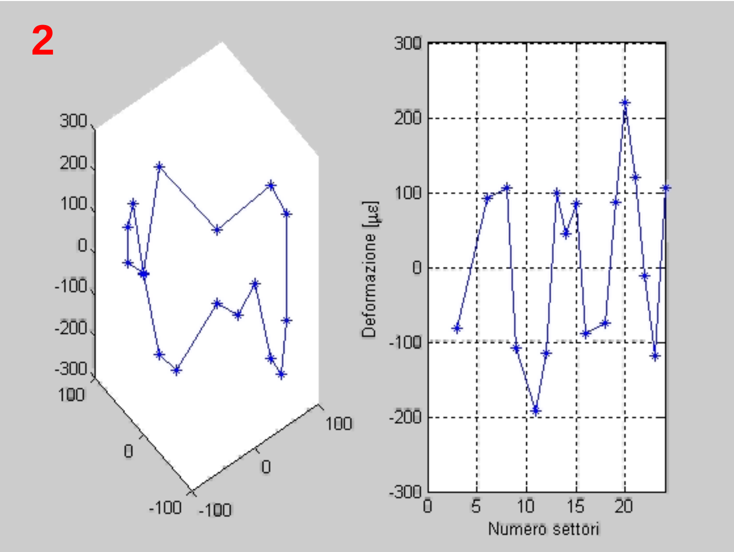


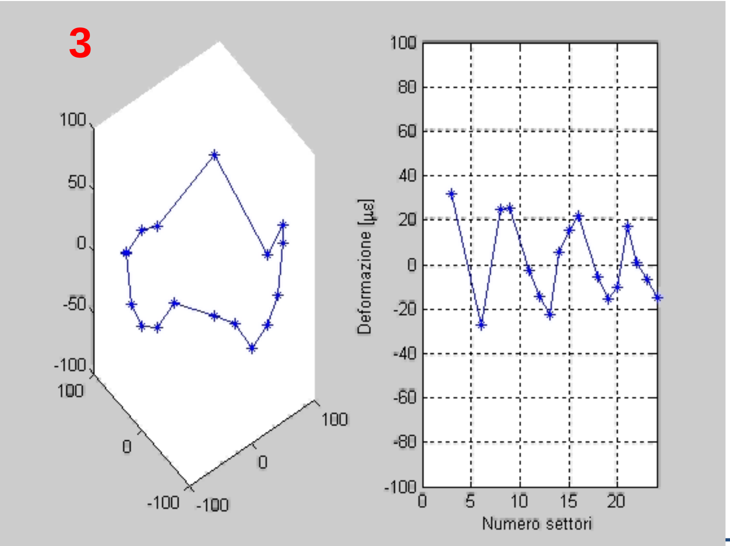
presence o two peaks at resonance

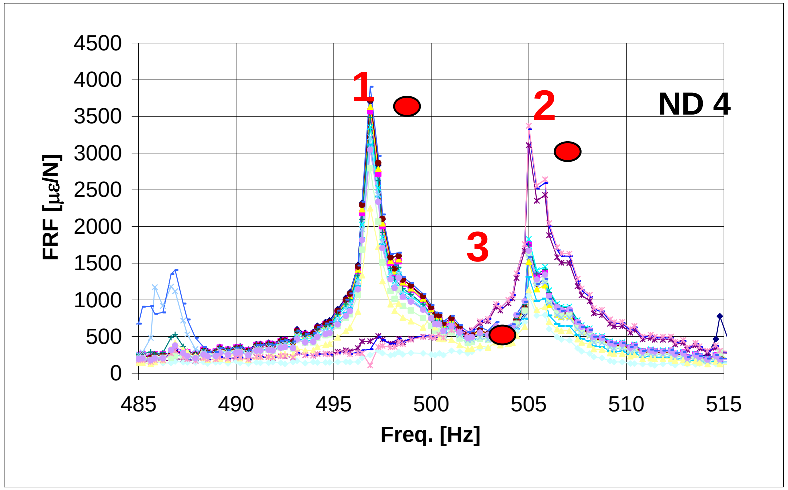
differences among the 24 sectors

Spinning test results – standing and rotating waves



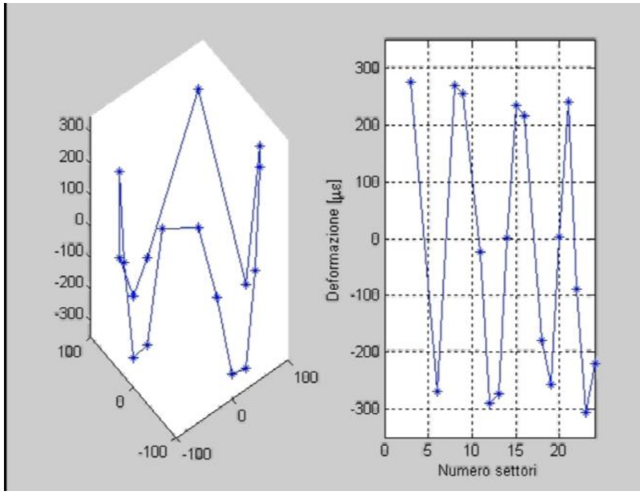




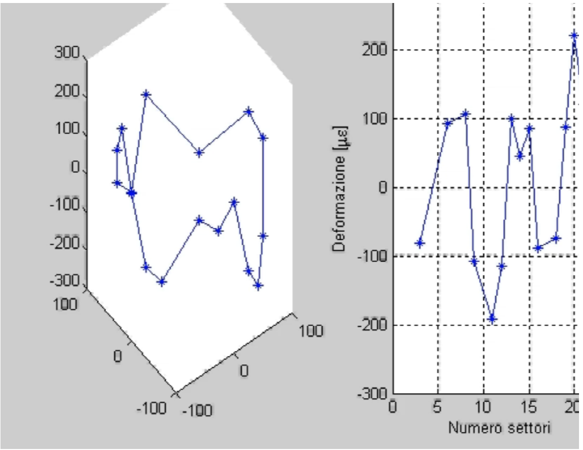


Spinning test results – standing and rotating waves

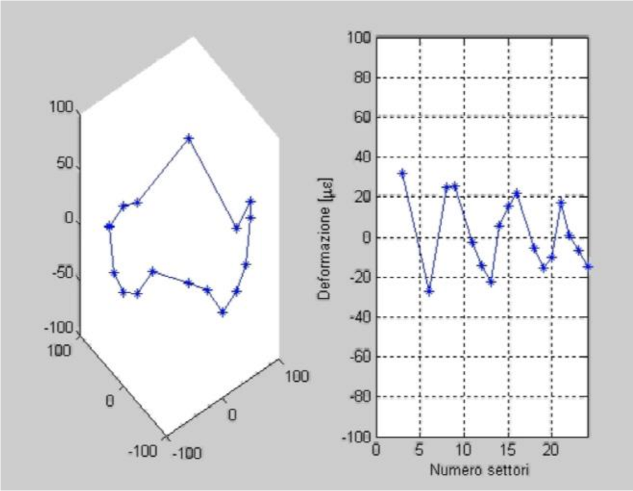
1

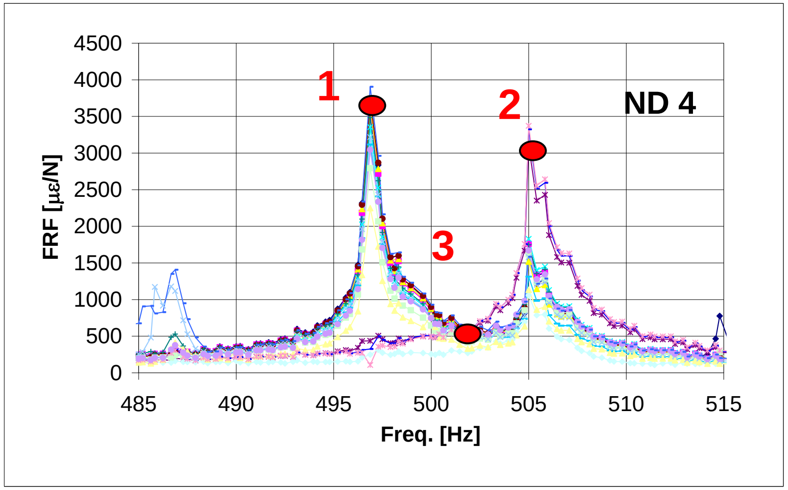


2

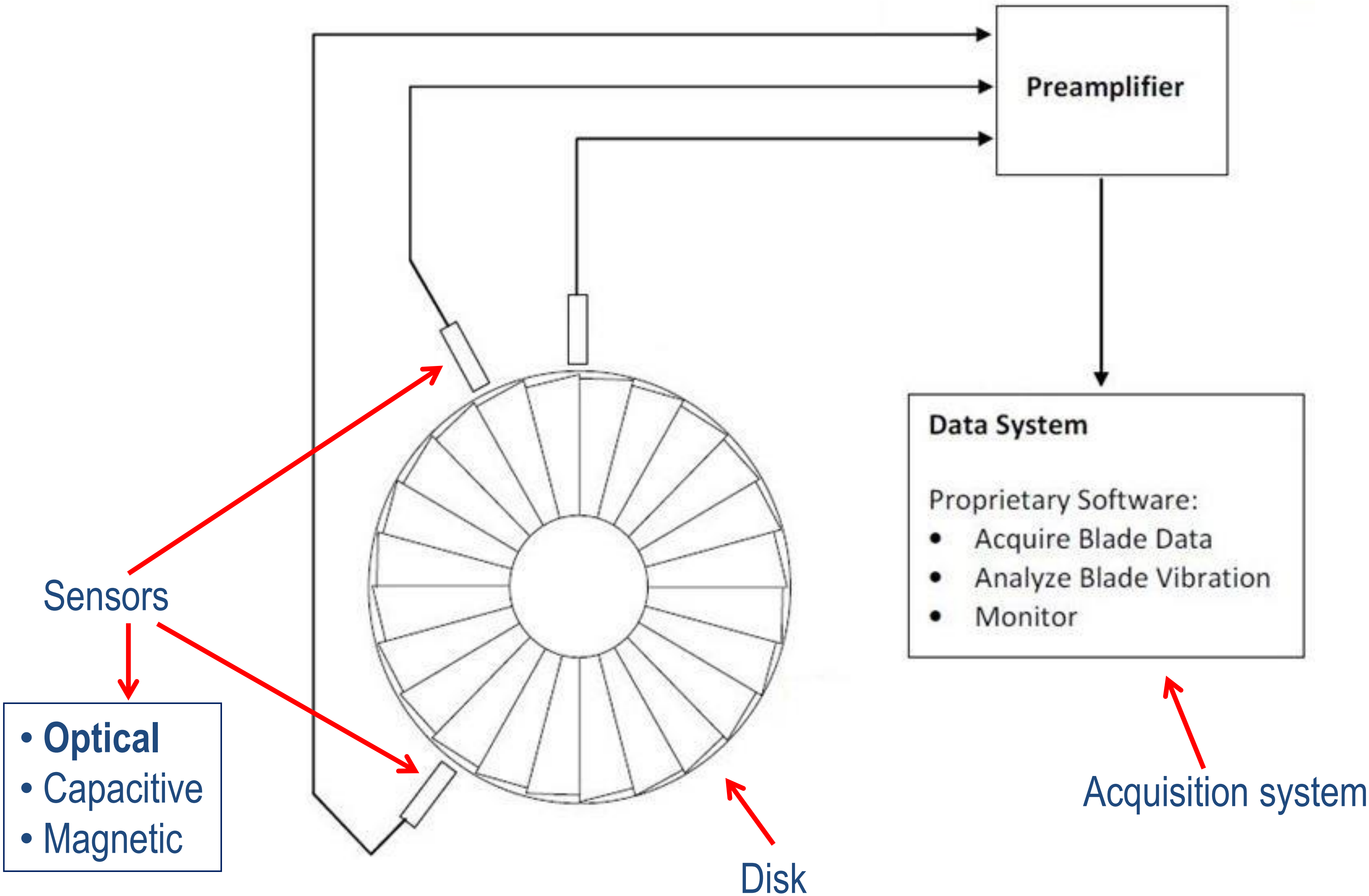


3





Blade Tip Timing Measurement System



Blade Tip Timing Measurement System

BTT vs Strain Gauges

1) **BTT is a Non-Intrusive Measurement system**
   - Strain gauges are attached on the blade

Affecting the blade dynamic behavior	If critical environment, fast strain gauges deterioration

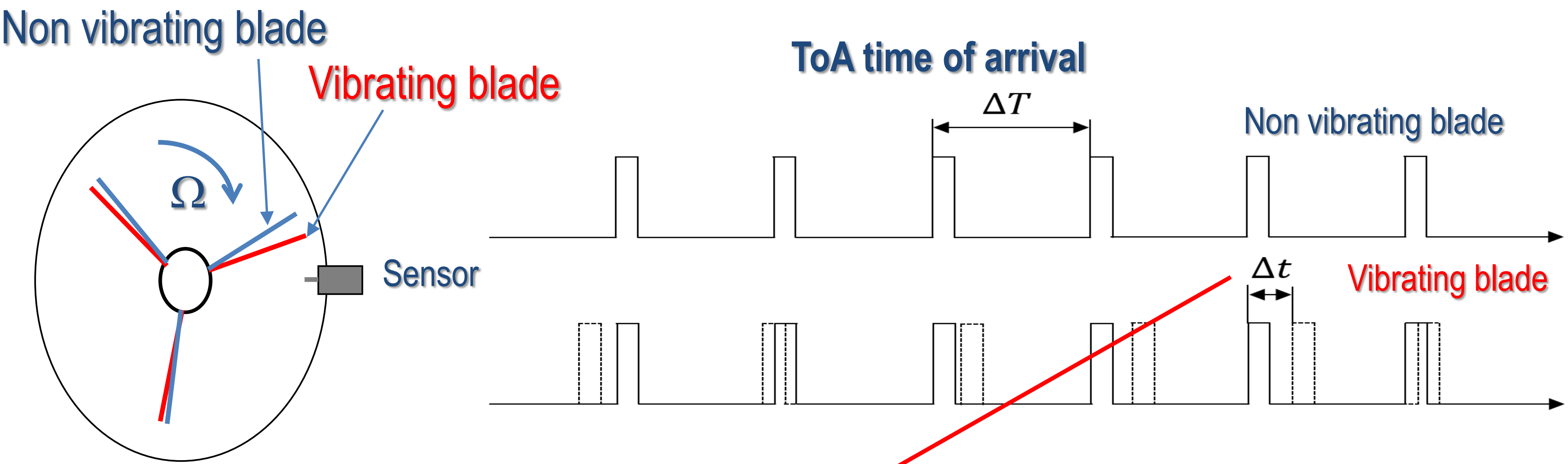
2) **Each sensor detects the vibration of each blade**
   - Instead the strain gauges are attached only to few blades

BTT main outputs

1. **Blade resonance frequency**
2. **Blade maximum vibration amplitude**

Blade Tip Timing Measurement System

Basic principle



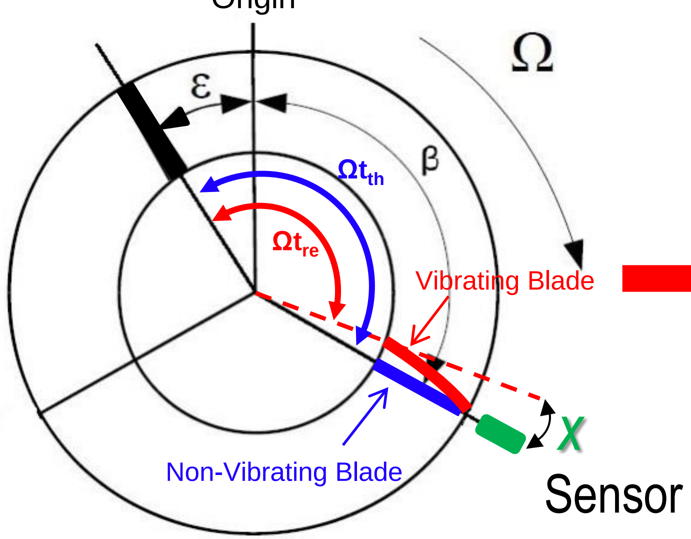
$x(\Omega)=\Delta t \cdot \Omega$

Vibration displacement at a given rotational speed Ω

Blade Tip Timing Measurement - Basic principle

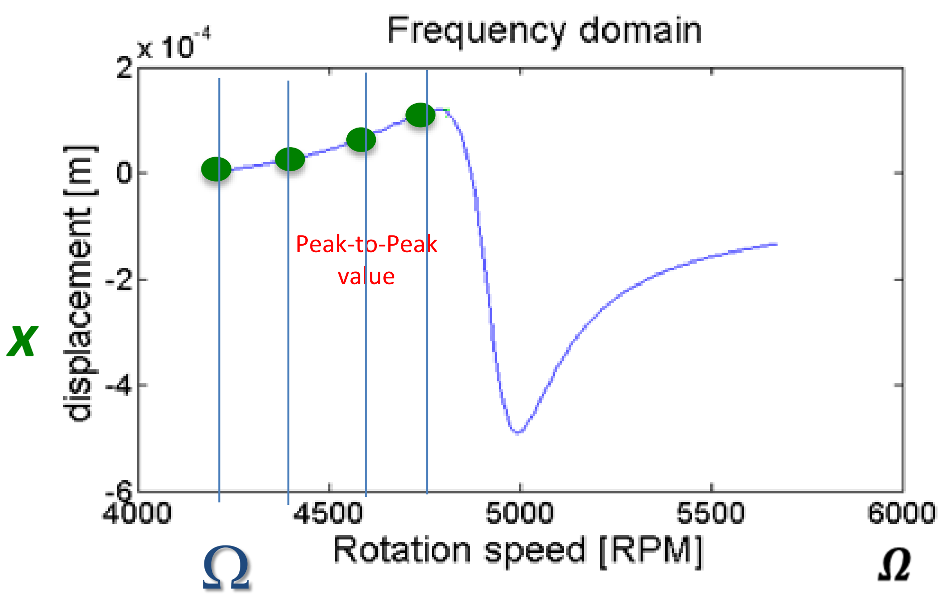
In presence of vibration the real ToA (t_{re}) could be greater or less than the theoretical ToA (t_{th}).

Blade at time t = 0



The vibrating blade reaches the sensor in advance or in delay with respect to the non-vibrating reference $t_{re}>t_{th}$ or $t_{re}<t_{th}$

$$x(\Omega)=\Delta t\cdot\Omega=(t_{re}-t_{th})\cdot\Omega$$



Blade Tip Timing Measurement - Basic principle - one sensor

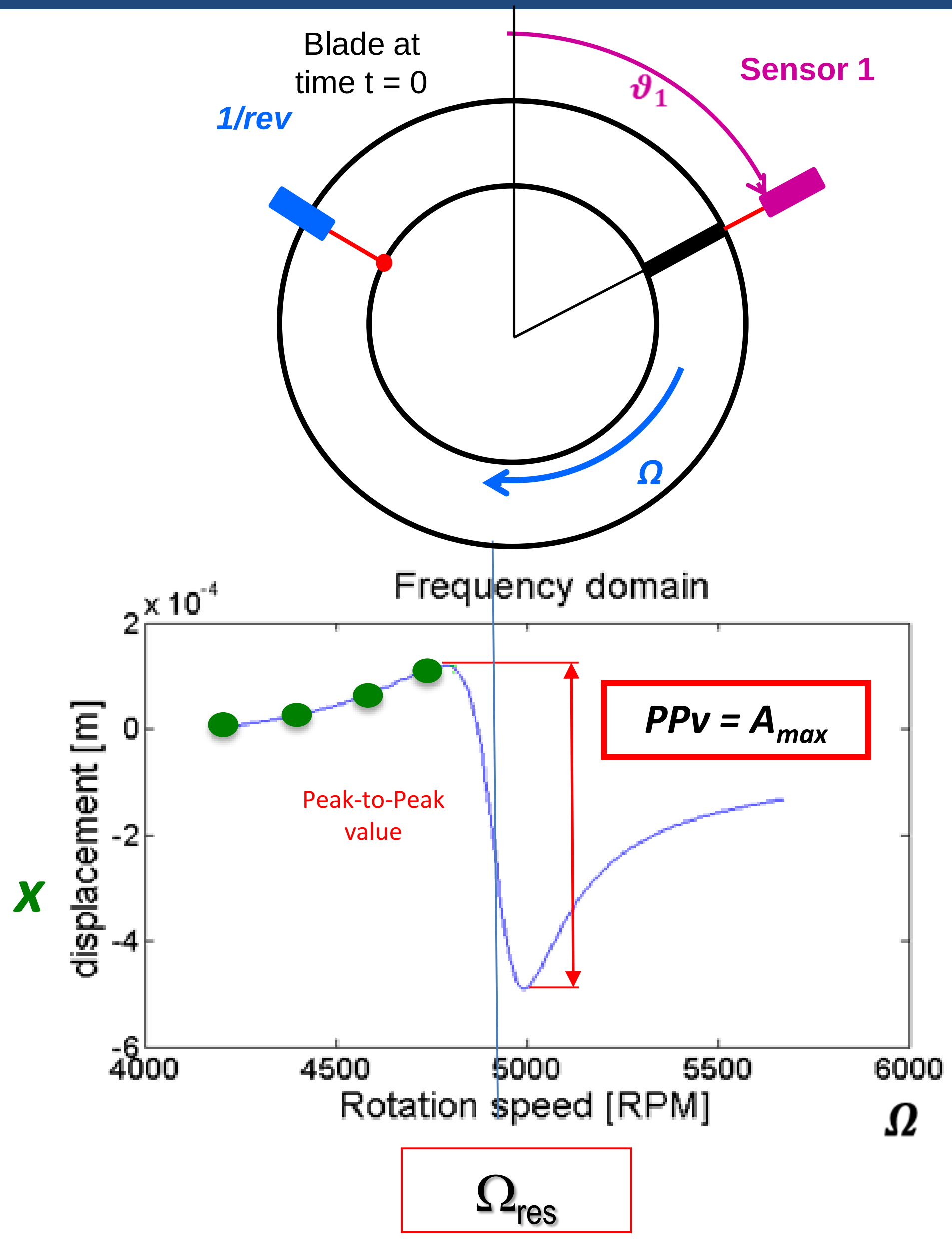
$$x(\omega) = A(\omega) \sin\left[EO \cdot \Omega t + \varphi(\omega)\right]$$

$$\omega = EO \cdot \Omega$$

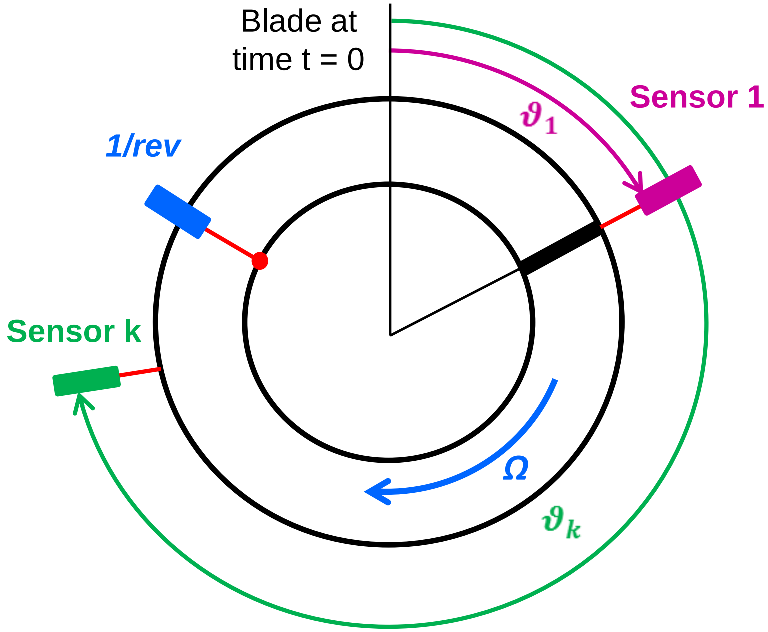
For a given probe 1

$\theta_1 = \Omega t_1$ **Angular position of the sensor with respect to the blade**

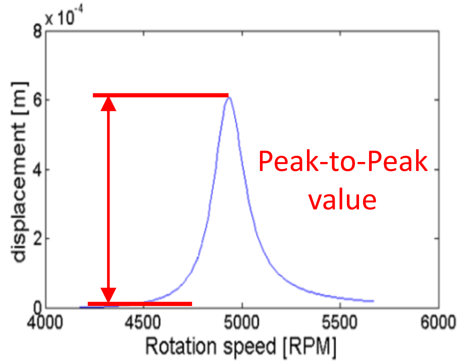
$$x(\omega) = A(\omega) \sin\left[EO \cdot \theta_1 + \varphi(\omega)\right]$$

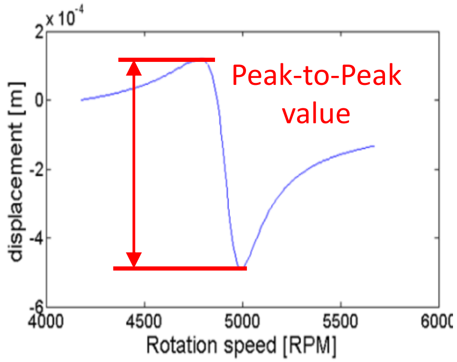


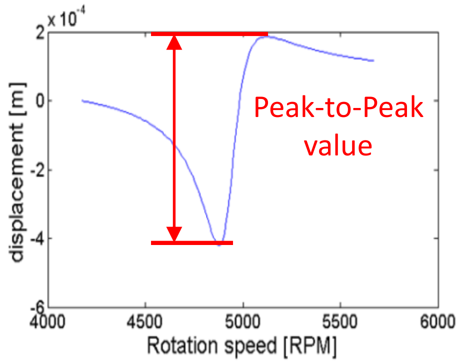
Blade Tip Timing Measurement System: Basic principle - more sensors



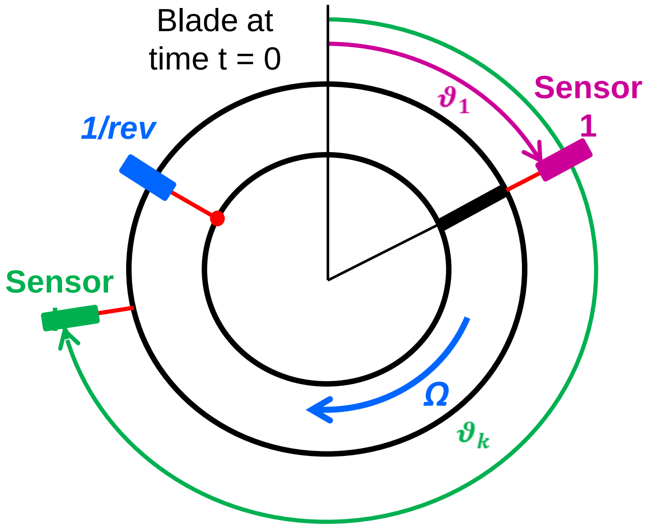
Depending on the position of the blade θ_k with respect with the sensor……





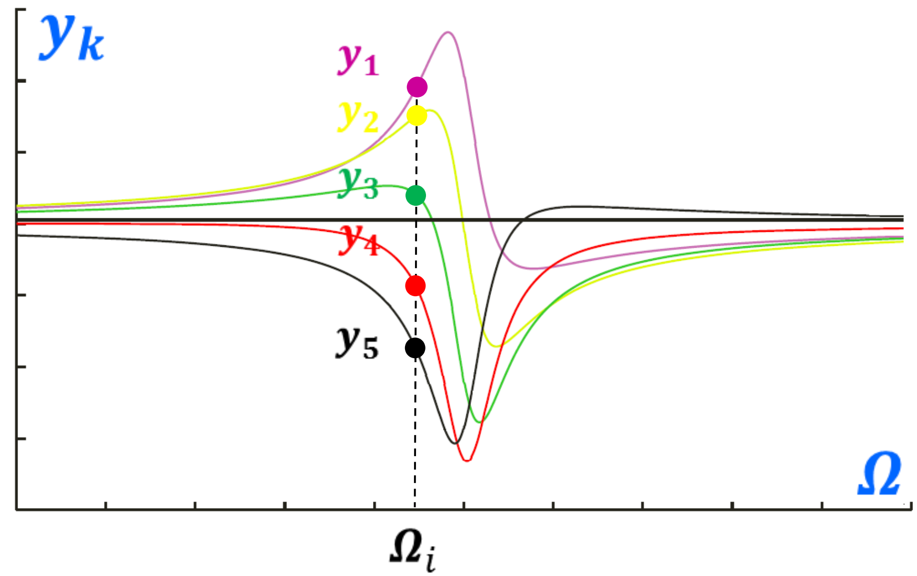


Blade Tip Timing Measurement System Basic principle - more sensors



$\theta_1=\Omega t_1$ Angular position of the sensor 1 with respect to the blade

Blade response detected by each sensor



Forced response of the blade

$$y_k(\omega) = c(\omega) + A(\omega)\sin[EO\vartheta_k + \varphi(\omega)]$$

k > 3

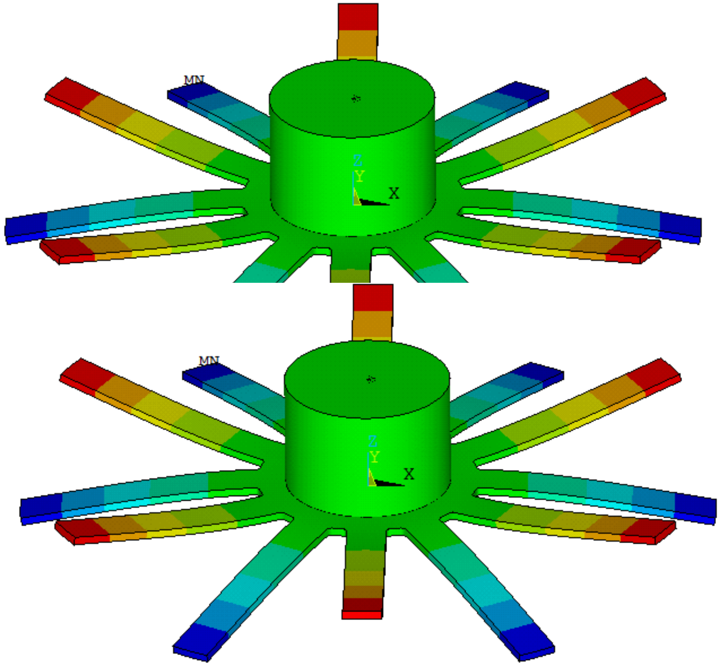
Least Square Problem

$c(\omega_i)$, $A(\omega_i)$, $\varphi(\omega_i)$

BTT – Measurement example

Disk

- Diameter: 440 mm
- Thickness: 5 mm
- Beam shaped beam
- Material: Aluminum
- Number of Blades: 12

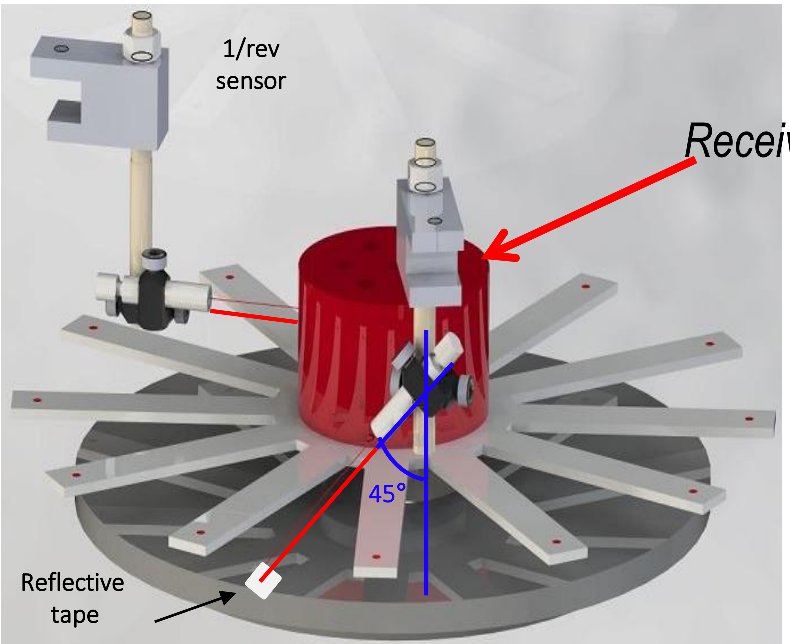


***Bending** out of plane mode shapes*

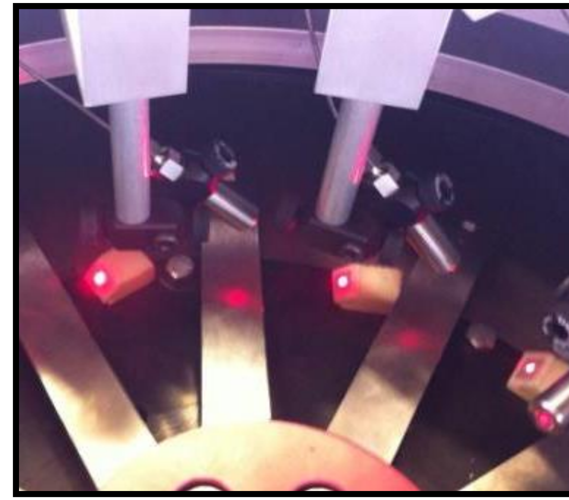
BTT – Measurement example - the spinning rig

BTT – Measurement example – BTT vs. Strain gauges

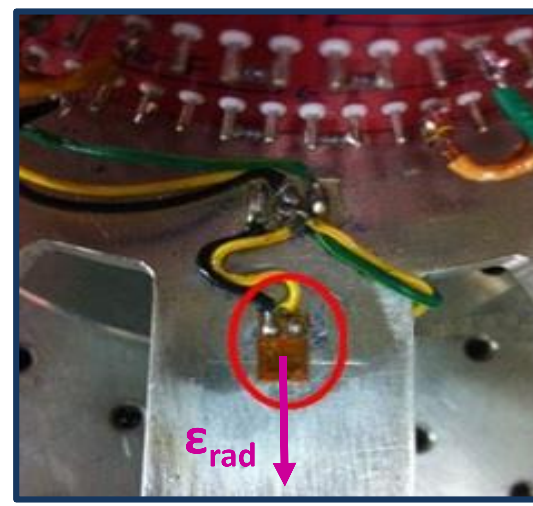
- Disk
- Excitation system: permanent magnets
- Blade Tip Timing measurement system (BTT)
- Strain gauges and telemetry system



BTT sensors

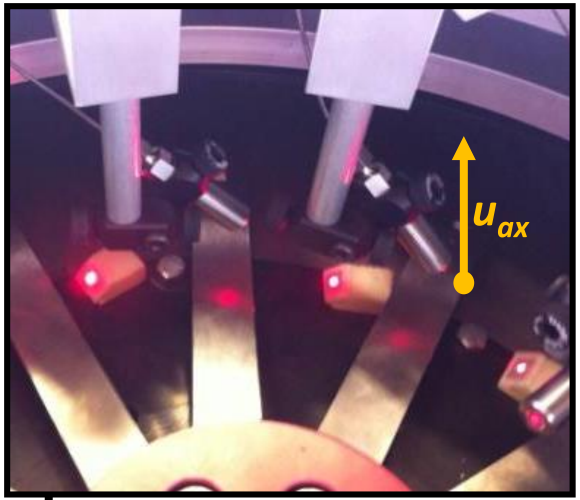


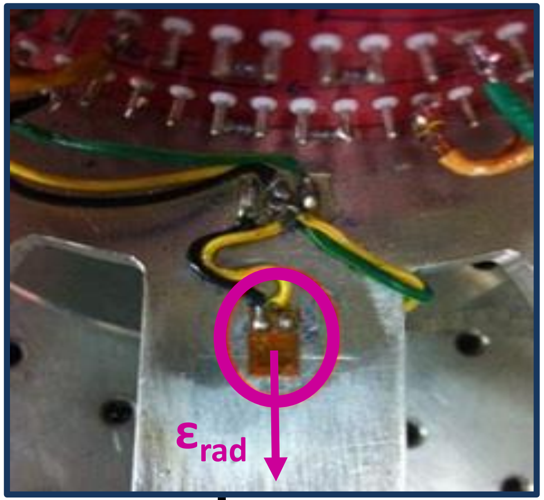
Strain Gauge

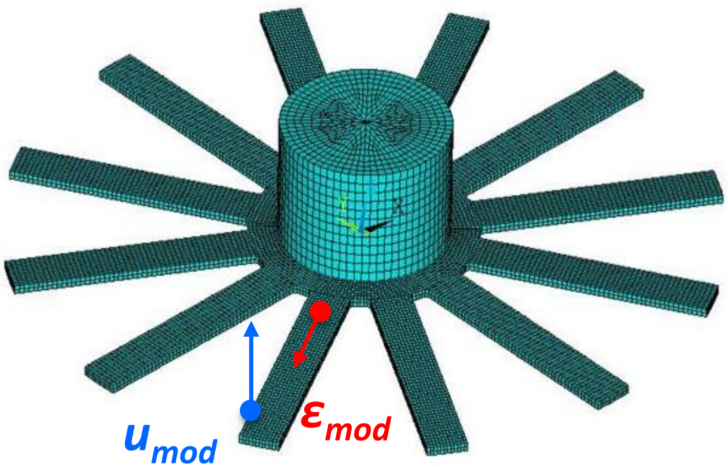


BTT vs. strain gauges

$A_{max} \approx 2000\ \mu m$







$u_{BTT} = u_{ax} \longleftrightarrow u_{SG} = K_{mod} \cdot \epsilon_{SG}$

$$K_{mod} = \frac{u_{mod}}{\varepsilon_{mod}}$$

Strain Gauges Reference

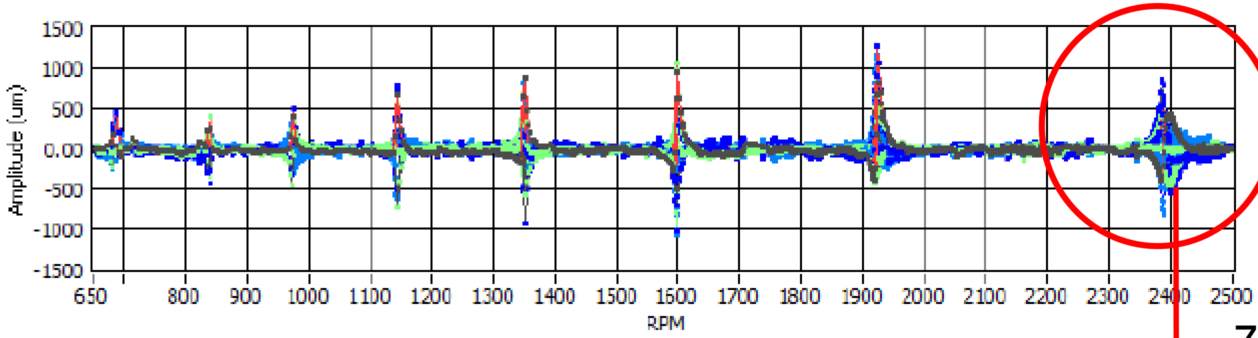
ND	f_{SG}	f_{BTT}	e_f	u_{SG}	u_{BTT}	e_u
-	Hz	Hz	%	μm	μm	%
5	158,0	158,0	**0**	1662,23	1631,48	**1,88**
6	159,3	160,2	**0,44**	2298,75	2275,60	**1,02**

$$e_f = \frac{|f_{SG} - f_{BTT}|}{f_{SG}} \cdot 100$$

$$e_u = \frac{|u_{SG} - u_{BTT}|}{u_{SG}} \cdot 100$$

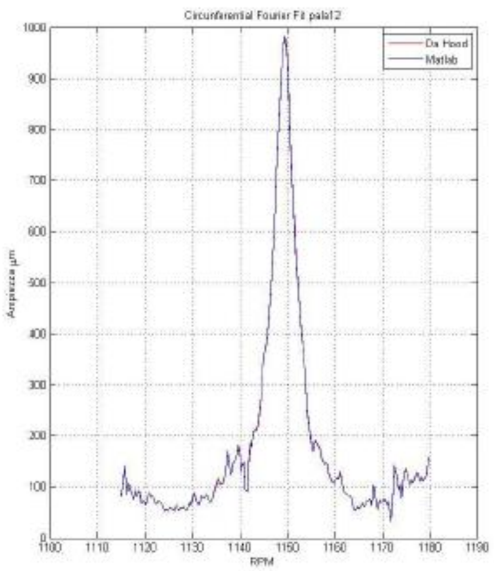
BTT – Measurement example – on a single blade

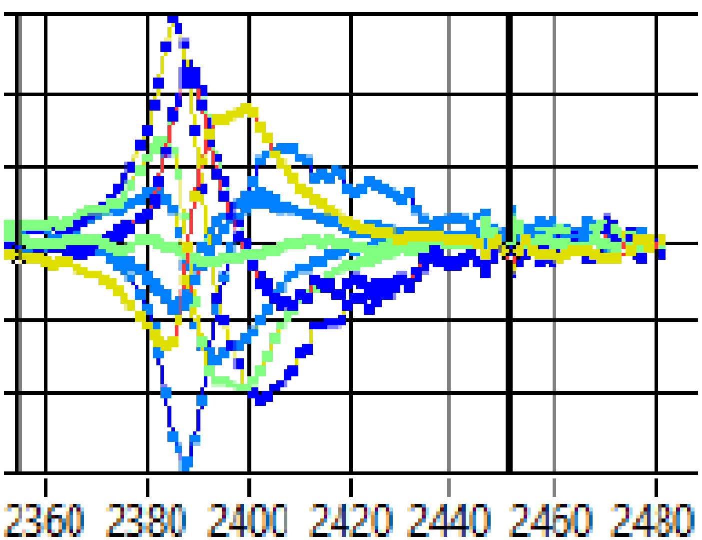
Signals for each blade



Zoom of the signals of the different sensors

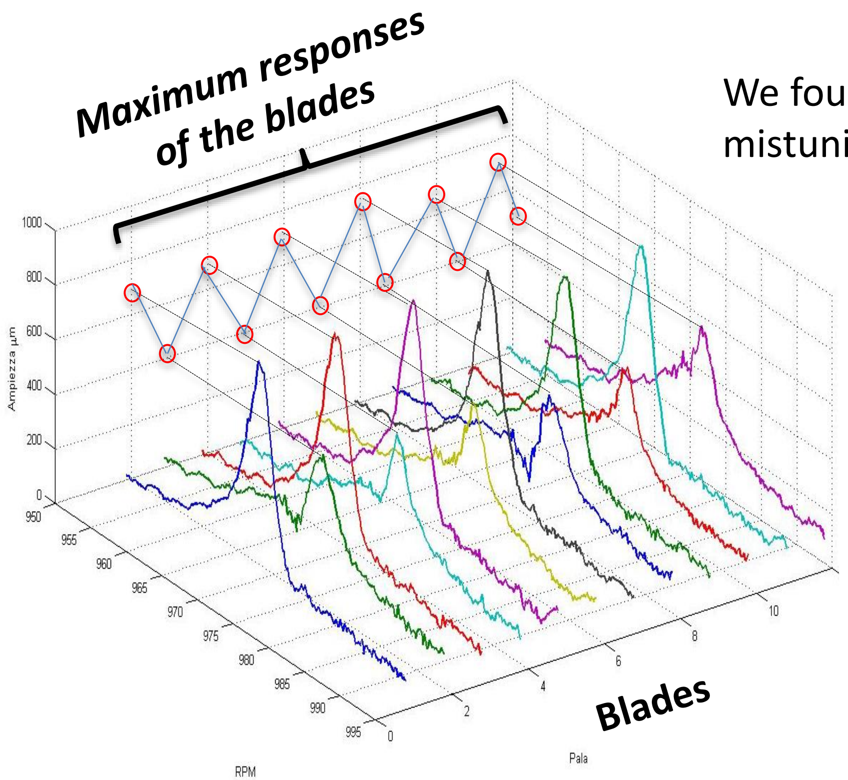
Reconstruction of the forced response of the blade





BTT- ND observation through the disk mistuning

Forced responses of the 12 blades



We found again a modulation of the response due to small mistuning

$$6 = 2 \cdot 3 \ (=ND)$$

$$ND = 3$$

The presence of small mistuning here could be used to find the nodal diameter from the tip timing measurements

Battiato, G..; Firrone, C.M.; Berruti, T. M. (2016) Forced response of rotating bladed disks: Blade Tip-Timing measurements. In: MECHANICAL SYSTEMS AND SIGNAL PROCESSING, vol. 85, pp. 912-926.